Figure 2.5: Time taken to reach the base of the giant branch as a function of stellar mass as given by eq. (2.4), shown against the detailed model points, for $Z = 0.0001$ and 0.03 which give the worst fit of all the metallicities. The maximum error over the entire metallicity range is 4.8% and the RMS error is 1.9%. The points for all $Z$ are not shown as this would clutter the presentation.

Note that $\mu$ is ineffective for $M < M_{\text{hook}}$, i.e. stars without a hook feature, and in this case the functions ensure that $x > \mu$.

Thus the time at the end of the MS, $t_{\text{MS}}$, and the time taken to reach the start of the GB (or end of the HG), $t_{\text{BGB}}$, are defined such that

$$t : 0.0 \rightarrow t_{\text{MS}} \quad \text{MS evolution}$$
$$t : t_{\text{MS}} \rightarrow t_{\text{BGB}} \quad \text{HG evolution.}$$

The starting values for $L$ and $R$ are the ZAMS points fitted by Tout et al. (1996). The next step is to fit the values at the end of the MS, $L_{\text{TMS}}$ and $R_{\text{TMS}}$, as well as at the end of the HG. The luminosity at the end of the MS is approximated by

$$L_{\text{TMS}} = \frac{a_{11} M^3 + a_{12} M^4 + a_{13} M^{a_{16} + 1.8}}{a_{14} + a_{15} M^5 + M^{a_{18}}} \quad (2.8)$$

with $a_{16} \approx 7.2$. This fit is fairly straightforward but the behaviour of $R_{\text{TMS}}$ is not so smooth and requires a more complicated function in order to fit it continuously. The resulting fit is

$$R_{\text{TMS}} = \frac{a_{18} + a_{19} M^{a_{21}}}{a_{20} + M^{a_{22}}} \quad M \leq a_{17} \quad (2.9)$$
$$R_{\text{TMS}} = \frac{c_1 M^3 + a_{23} M^{a_{26}} + a_{24} M^{a_{26} + 1.5}}{a_{25} + M^5} \quad M \geq M_* \quad , \quad (2.9a)$$