WEEK 7: ASTROPHYSICAL PHENOMENOLOGY AND BINARY STARS AS GW SOURCES

Lecture 12 by Phinney

Recommended Reading:

There are no good textbooks or even review articles that cover all, or even most of what Phinney presented in his lecture; but some important portions of it are covered in texts. Given the paucity of good references, students may want to *review the video version of Phinney's lecture*, which is on the web at the course web site. In addition, the following are recommended:

1. Optical luminosity density in the universe [a key input for estimating gravitational wave strengths], and Olber's paradox : pages 119–131 of P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, 1993).
2. Stellar evolution, degenerate remnants of stellar evolution (white dwarfs, neutron stars, black holes), and close binary systems [astrophysical phenomenology, not fundamental theory]: See most any textbook on modern astrophysics. For example, Bradley W. Carroll and Dale A. Ostlie, *Introduction to Modern Astrophysics*, (Addison-Wesley, 1996), chapters 13, 15 and 17. Note: two copies of this are on reserve in Millikan Library and one is in Robinson Library (the astronomy library) as a reference book, not to be taken out. Call number: QB461 .C35 1996.

Possible Supplementary Reading:

3. Shaun Cole et al, "The 2dF galaxy redshift survey: near infrared galaxy luminosity functions," Monthly Notices of the Royal Astronomical Society, 326, 255 (2001). Available at http://xxx.lanl.gov/abs/astro-ph/0012429 . In his lecture, Phinney talked about Figure 19, "Observational estimates of the star formation history of the Universe."
4. P. Kroupa, "Inverse dynamical population synthesis and star formation", Monthly Notices of the Royal Astronomical Society, 277, 1491 (1995). Available at http://xxx.lanl.gov/abs/astro-ph/9508117 . Phinney's handout shows Figures 1 and 2: histograms of observational data on periods, mass ratios and eccentricities of binaries.
5. In connection with Phinney's Lecture 13 this coming Monday: The following references on the evolution of binary star systems and black holes:
   a. Ronald E. Taam and Eric L. Sandquist, "Common Envelope Evolution of Massive Binary Stars," *Annual Reviews of Astronomy and Astrophysics 2000*, **38**, 113 (2000). Available to Caltech users and others whose institutions have subscriptions, at http://astro.AnnualReviews.org/ .
   b. G.E. Brown, C.H. Lee, R.A.M.J. Wijers, and H.A. Bethe, "Evolution of Black Holes in the Galaxy," *Physics Reports*, **333**, 471 (2000). Available on the web at http://xxx.lanl.gov/abs/astro-ph/9910088 .

c. P.J. Armitage and M. Livio, "Black hole formation via hypercritical accretion during common-envelope evolution, *Astrophysical Journal*, **532**, 540 (2000). Available at http://xxx.lanl.gov/abs/astro-ph/9906028 , and at the Astrophysical Journal web site, http://www.journals.uchicago.edu/ApJ/ .

d. A.R. King and M.C. Begelman, "Radiatively-driven outflows and avoidance of common-envelope evolution in close binaries," *Astrophysical Journal Letters*, **519**, L169 (1999). Available at http://xxx.lanl.gov/abs/astro-ph/9904105 , and at the Astrophysical Journal web site, http://www.journals.uchicago.edu/ApJ/ .

Assignment, to be turned in at beginning of class on Wednesday 20 February by students registered in the course:

A. State what reading you have done, related to the course, during this past week.

B. Work those exercises, from the list below, that are useful for you (i.e. that are at the appropriate level for you [neither much too hard nor too easy] and that have a ratio of grunge to learning that is reasonable.

C. If A. and B. do not constitute enough to have taught you a reasonable amount about this week's topic, then do one or more of the following:
   i. If you already know a lot about this week's topic, just say so and stop.
   ii. Invent your own exercises and work them.
   iii. Carry out further reading and state what you have done.
   iv. Seek private tutoring from a knowledgable person about this week's topic.
   v. Pursue some other method of learning about this week's topic, and state what you have done.

EXERCISES

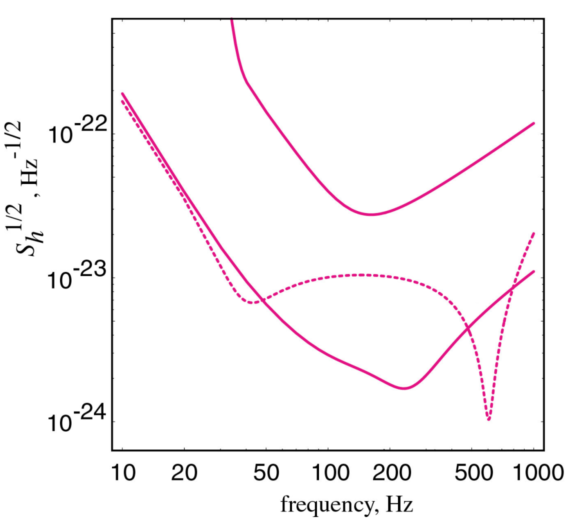
Exercises related to Phinney's Lecture 12

1. **Rates for Compact Binary Inspirals as seen by LIGO**
   The rate of neutron-star / neutron-star (NS/NS) inspirals and mergers in our galaxy is estimated, from the statistics and selection effects of searches for binary pulsars, to be $R_{\rm gal} = 10^{-6}$/yr; and the rate for black-hole / black-hole (BH/BH) inspirals is estimated, from (evolutionary) population synthesis arguments to be between 10^{-7} and 10^{-5}/yr.
   a. What are the rates per unit volume in the entire universe?
   b. The noise curves for LIGO's initial and advanced interferometers were shown and discussed in slide 24 of Kip's first lecture in this course (on the course web site). Below is a reproduction of the noise curves. The solid, broad-band curves are the relevant ones for the inspiral waves, since the waves sweep through a broad range of frequencies. (The dotted, narrow-band curve is for an interferometer that is tuned to go after waves from Low-Mass X-Ray Binaries near 600 Hz.) The upper solid curve is for an initial 4km LIGO interferometer and the lower is for an advanced (LIGO-II) inteferometer. For each of these interferometers,

estimate the distance to which the waves can be seen from NS/NS inspiral and from BH/BH inspiral, assume reasonable values for the masses of the NS's and BH's, and explain why you chose those values.

[Hint: In Exercise 5 of Week 5 you derived the gravitational wave fields h_+ and $h_\times$ for a binary, as a function of the binary's masses, distance, and frequency; and in Exercise 4 of Week 6 you derived a formula for the rate of inspiral $a(t)$ for the binary, from which using Kepler's laws you can get the wave frequency $f(t)$. Knowing $f(t)$, you can compute the number of gravitational-wave cycles $n_{\rm cyc}$ that the binary emits as it sweeps through a bandwidth equal to frequency, $\Delta f = f$, near the noise curve's minimum. The quantity $h_c = \sqrt{h_+^2 + h_\times^2}\sqrt{n}$ is called the waves' characteristic amplitude; it is the amplitude of the signal built up in the detector, as the waves sweep through the band Δf. The detector's rms noise that masks this signal is $h_{\rm rms} = \sqrt{5 \Delta f S_h(f)}$, where the factor 5 takes account of the fact that the source is at some random location on the sky rather than at the optimal location, directly overhead or underfoot. The signal to noise ratio is $S/N \simeq h_c/h_{\rm rms}$. Assume a reasonable S/N for confident detection.]

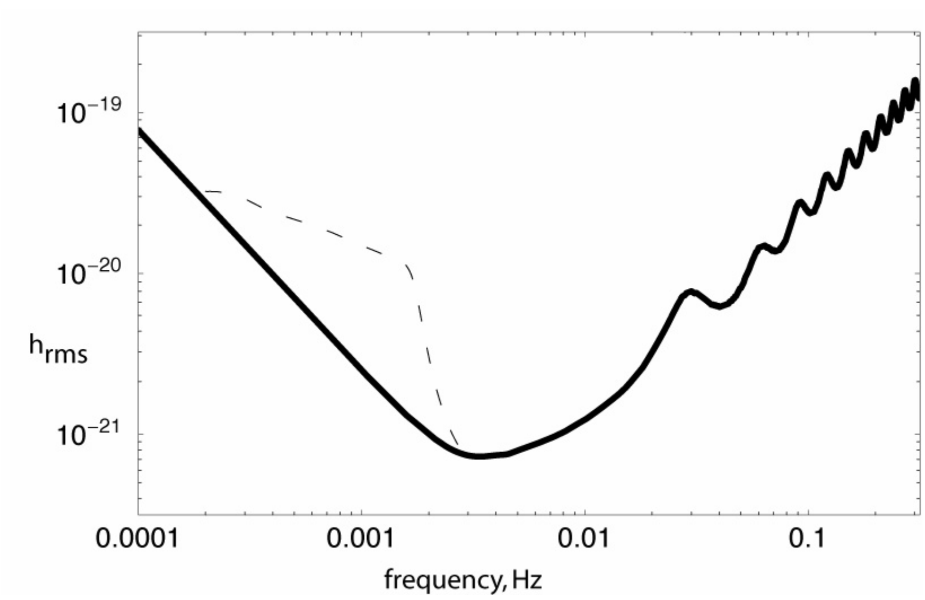
c. Combine the distances to which the waves can be seen (part b) with the event rate in the universe (part a) to estimate the number of NS/NS and BH/BH inspirals that will be seen per year, by LIGO-I and LIGO-II interferometers.



2. **Gravitational Waves from White Dwarf Binaries in our Galaxy**

Idealize all white-dwarf / white-dwarf (WD/WD) binaries in our galaxy as having masses of $1M_\odot/1M_\odot$ and circular orbits, and assume that they have been created in our galaxy, through the evolution of main-sequence binaries, at a rate of one WD/WD binary every ten years, for the past 10^{10} years (roughly the current age of the galaxy). Assume, further, that they are created with a distribution of initial binary separations a_o that is uniform in the log, i.e., $dN/d\ln a_o\, dt =$constant, for separations ranging from 0.1pc (0.3 light years) — larger than which the binary will be fairly easily disrupted by passing stars — and the separation at which the stars' surfaces touch.

a. What is the gravitational-wave frequency f_1 below which gravitational radiation reaction has not substantially influenced a WD/WD binary's separation a, and below which it has strongly influenced the separation during the life of the galaxy. [Hint: See Exercise 6 for Week 6.]
b. Above frequency f_1, what is the number of WD/WD binaries in our galaxy today, per unit log of gravity-wave frequency, $dN/d\ln f$?
c. Below f_1, what is $dN/d\ln f$? [Hint: Write down a conservation law for binaries in frequency space, analogous to that for charge in spacetime, but with a source term dictated by the assumptions given above.]
d. When one searches for the gravitational waves from WD/WD binaries by integrating up LISA's data over a time of $\hat{\tau} = 1$ year, one should be able to distinguish individual binaries from each other, if their separation in frequency exceeds $\Delta f \simeq 1/\hat{\tau}$. What is the frequency f_2 above which individual binaries can be distinguished and below which they cannot?
e. Below frequency f_2 the WD/WD binaries produce gravitational-wave "noise" against which LISA must search for waves from other sources, e.g. from BH/BH binaries. Estimate the magnitude of that noise as a function of frequency, expressed in terms of $h_{\rm rms}$, the root-mean-square value of h_+ and $h_\times$ in a bandwidth equal to frequency. Your answer should agree, roughly, with the dashed WD/WD noise curve that is shown, together with LISA's planned noise curve, in the following figure:



Exercises related to Phinney's Lecture 13, next Monday

3. **Common-Envelope Binary Evolution**
The following is an example of a process that occurs in the evolution of close binary systems from their initial, main-sequence configurations to final compact-body configurations (WD/WD, NS/NS, BH/BH, etc): A 16 solar mass star, late in its life, evolves into a red giant phase — which has a $4M_\odot$ compact core and a $12M_\odot$ puffed-up envelope. The radius of the envelope is $R \simeq 200R_\odot$, but most of its mass is contained

inside $R_1 \simeq 100 R_\odot$, which you can idealize as having uniform density. Suppose that this star has a $3M_\odot$ companion, and when the big star expands into its red giant phase, the outer part of its envelope engulfs this companion. As the companion orbits inside the giant star's envelope, it stirs the envelope up, gradually feeding the orbital energy into the envelope's gas, causing gas to get ejected and causing the companion to slowly spiral inward. Ultimately the companion reaches a radius small enough that *the energy it has injected into the giant's envelope is enough to eject all of the envelope's mass*. The result, then, is a $3M_\odot$ companion orbiting a $4M_\odot$ compact star (the red giant's remnant core). Estimate the separation between these two stars.

4. **Eddington Limit**

A compact star accretes gas from a dense surrounding medium (e.g., in the common-envelope evolution described above). Idealize the accretion as spherically symmetric. When the gas hits the surface of the compact star, its energy of infall is converted into heat and thence into outgoing radiation. This radiation flows out through the accreting gas and, if the gas was not already highly ionized, the radiation ionizes it. It is a good approximation to treat the infalling gas as fully ionized. Then the radiation can scatter off electrons (with scattering cross section $(8\pi/3)r_o^2 = 0.665 \times 10^{-24}$cm, where r_o is the classical electron radius. In this scattering, on average, all of the photon's momentum is transferred to the electron, giving the electron an outward kick. The collective influence of all these kicks on electrons, gives a net outward force per unit mass on the infalling gas.

a. Derive a formula for the net outward force per unit mass of gas in terms of the gas's mass density and the outflowing flux of radiation energy $\mathcal{F}$.

b. Explain why the radiation's similar outward force on the protons, per unit mass of gas, is negligible compared to that on the electrons.

c. Explain why the inward force of gravity acting on the protons, per unit mass of gas, is far larger than that acting on the electrons.

d. With the outward radiation force acting on the electrons and the inward gravitational force acting on the protons, a charge separation develops between the electrons. This charge separation produces an electric field that locks the electrons and protons together. Estimate very roughly how much charge separation occurs and show that it is totally negligible compared to the size of the star.

e. Because the outflowing radiation flux is $\mathcal{F} = L/(4\pi r^2)$ where r is radius and L is the star's luminosity, produced by accretion, the outward radiation force per unit mass on the gas scales as $1/r^2$ — the same scaling as the gravitational force per unit mass. Therefore, there is a critical luminosity $L_{\rm Edd}$ (called the *Eddington Luminosity*) above which the radiation force overwhelms the gravitational force, turning off the accretion, and below which the accretion can proceed. Derive a formula for $L_{\rm Edd}$. If the gas is pure hydrogen, show that the numerical value of $L_{\rm Edd}$ is $1.38 \times 10^{38}(M/M_\odot)$ergs/sec$= 3.5 \times 10^4 (M/M_\odot) L_\odot$, where M is the mass of the accreting star. What is $L_{\rm Edd}$ for pure helium gas?

e. It is reasonable to expect that in a steady state most all of the kinetic energy of the infalling gas, after hitting the star's hard surface, gets converted into outflowing radiation. Explain why. What, then, is the accretion rate $\dot{M}_{\rm Edd}$ that produces the Eddington Luminosity. Express your answer in terms of the star's mass M and radius R. Evaluate your answer numerically, in solar masses per year, for the sun, a white dwarf, and a neutron star. Any accretion at a rate higher than this is called *hypercritical accretion*.

f. If the gas surrounding the star is sufficiently dense, then it is reasonable to expect that the accretion rate will get buffered into the Eddington rate $\dot{M}_{\rm Edd}$. Explain why.

h. For a black hole, which does not have a solid surface, hypercritical accretion can occur. Suppose that the efficiency for conversion of rest mass of infalling gas into outflowing radiation energy is ϵ, and that the outflowing radiation gets buffered into the Eddington value $L_{\rm Edd}$. What is the time required for the hole's mass to double? Assume that the gas is all hydrogen.