Ph 237a: Gravitational Waves 9 January 2001

WEEK 1: OVERVIEW

Recommended Reading:

Note: Almost all readings will be available for downloading on the web; the url will be given at the end of each reference.

1. Scott A. Hughes, Szabolcs Marka, Peter L. Bender and Craig J. Hogan, "New physics and astronomy with the new gravitational-wave observatories", to be published in Proceedings of the 2001 Snowmass Meeting, *http://xxx.lanl.gov/abs/astro-ph/0110349*.

Possible Supplementary Reading [listed in reverse chronological order]:

2. Kip S. Thorne, "The scientific case for advanced LIGO interferometers", LIGO Document Number P-000024-00-D, *available as a pdf file on the Ph237 web site.*
3. Barry C. Barish and Rainer Weiss, "LIGO and the detection of gravitational waves", Physics Today, 52, 44 (October 1999), *not available electronically*
4. Barry C. Barish, "The detection of gravitational waves with LIGO", Proceedings of DPF'99, *http://xxx.lanl.gov/abs/gr-qc/9905026*
5. Bernard F. Schutz, "Gravitational wave astronomy", Classical and Quantum Gravity, 16, A131-A156 (1999), *http://xxx.lanl.gov/abs/gr-qc/9911034*
6. Eanna E. Flanagan, "Sources of gravitational radiation and prospects for their detection", Proceedings of GR15, *http://xxx.lanl.gov/abs/gr-qc/9804024*
7. Kip S. Thorne, "Probing black holes and relativistic stars with gravitational waves", in Black Holes and Relativistic Stars, Proceedings of a Conference in Memory of S. Chandrasekhar, ed. R. M. Wald (University of Chicago Press, Chicago, 1998), pp. 41-78. *http://xxx.lanl.gov/abs/gr-qc/9706079*

Assignment, to be turned in at beginning of class on Wednesday 16 January by students registered in the course:

A. State what reading you have done, related to the course, during this past week.
B. Work those exercises, from the list below, that are useful for you (i.e. that are at the appropriate level for you [neither much too hard nor too easy] and that have a ratio of grunge to learning that is reasonable.
C. If A. and B. do not constitute enough to have taught you a reasonable amount about this week's topic, then do one or more of the following:
   i. If you already know a lot about this week's topic, just say so and stop.
   ii. Invent your own exercises and work them.
   iii. Carry out further reading and state what you have done.
   iv. Seek private tutoring from a knowledgable person about this week's topic.
   v. Pursue some other method of learning about this week's topic, and state what you have done.

EXERCISES

Note: Work only those exercises that are useful for you!

1. **Multipolar expansion of the gravitational-wave field**
   Fill in the details of the argument sketched on slide 10 of Kip's lectures. In particular, show that each of the terms in the expansion of h is dimensionless, and explain why the terms could not have any other form.

2. **Strengths of the waves for various multipoles**
   a. Give an order of magnitude formula for the contribution of each multipole to the gravitational-wave field h — a formula analogous to that derived on slide 11 for the mass quadrupole moment. Express your answer in terms of the source's mass and internal velocity, and the distance to the source.
   b. Assuming that the internal velocity is generated by the source's own gravitational field, give an alternative answer in terms of the source's mass and radius, and the distance to the source.
   c. Compare with similar order of magnitude formulas for the contributions of the electric multipoles and magnetic multipoles to electromagnetic radiation.
   d. What is the magnitude (a dimensionless number) of the multipole's contribution to h in the case of colliding black holes at a distance of 100Mpc (about 300 million light years)? In the case of a neutron-star binary (with neutron-star masses equal to 1.4 that of the sun) at 100Mpc distance when the emitted waves have the frequency at which LIGO's noise is smallest?
3. **How LIGO works**
   On slide 13 of Kip's lectures there is a graph in the lower left corner that he did not discuss. Explain what it means and use it to explain in some detail how a LIGO interferometer works.
4. **Standard Quantum Limit for advanced LIGO interferometers**
   On slide 25 of Kip's lectures there is a statement that the advanced interferometers in LIGO will monitor the motions of their 40kg sapphire mirrors with an accuracy about the half width of the mirror's quantum (Schroedinger) wave function. This accuracy is called the "standard quantum limit" because, for conventional interferometer designs (such as that of the first LIGO interferometers), quantum mechanics prevents an accuracy better than this from being achieved.
   a. Derive a formula for the half width of the wave function in terms of Planck's constant, the mass of the mirror, and the frequency of the gravitational waves (which is approximately $f \sim 0.5/$(time during which each measurement of the mirror's center-of-mass position is made).
   b. Evaluate your formula numerically, for the frequency of the minimum of the advanced-detector noise curve, and thereby show that the detector does, indeed, operate near the standard quantum limit.
5. **Relative motions of LISA's spacecraft**
   LISA's spacecraft have an orbit shown in slide 27 of Kip's lectures. This orbit is disturbed by the gravitational fields of the planets, most especially Jupiter. Estimate

the relative motion of the spacecraft induced by Jupiter, and compare with the relative motion quoted in Kip's slide 27.