Ph 237b: Gravitational Waves 22 May 2002

WEEK 18: LISA's Disturbance Reduction System and Time Delay Interferometry

Lecture 33 by Bonny Schumaker (JPL) [Disturbance Reduction System]
Lecture 34 by John Anderson (JPL) [Time Delay Interferometry]

Reading Related to These Lectures:

Items in bold are recommended; others are supplementary.

1. **B.L. Schumaker, "Overview of Disturbance Reduction Requirements for LISA," (JPL document, 01 May 2002)**; available on the course web site. This corresponds fairly well to the material that Schumaker presented in her Lecture 33.
2. References on Time-Delay Interferometry for LISA. These are all on the course web site.
   - (a) M. Tinto, ”Time Delay Interferometry”, CaJAGWR lecture (April 2002). This was a Caltech CaJAGWR lecture covering much the same material as Armstrong's lecture. Video and viewgraphs for the lecture are available on the web at: http://www.cco.caltech.edu/ cajagwr/scripts/seminars.html .
   - (b) Reference AET1: Tinto, M., and Armstrong, J. W. 1999 ”Cancellation of Laser Noise in an Unequal-Arm Interferometer Detector of Gravitational Radiation”, Phys. Rev. D, 59, 102003. (This shows how to do time-domain cancellation of laser noise in an unequal-arm LISA Michelson interferometer, establishes the required armlength knowledge, and compares with approximate laser-noise cancellation using Fourier techniques.)
   - (c) Reference AET2: Armstrong, J. W., Estabrook, F. B., and Tinto, M. 1999 ”Time-Delay Interferometry for Space-Based Gravitational Wave Searches”, ApJ, 527, 814. (Develops general TDI-combinations for un-equal arm interferometers and gives method for computing sensitivity of each. Transfer functions are for single proof mass/spacecraft and single laser/spacecraft.)
   - (d) Reference AET3: **Estabrook, F. B., Tinto, Massimo, and Armstrong, J. W. ”Time-Delay Analysis of LISA Gravitational Wave Data: Elimination of Spacecraft Motion Effects”, Phys. Rev. D. 62, 042002.** (Generalizes AET2 above, using transfer functions for baseline LISA configuration: unequal armlengths, 2 lasers/spacecraft and 2 proofmasses/spacecraft. Lasers assumed to have same center frequencies and no orbital Doppler shifts. Derives new 4-link TDI-combinations.)
   - (e) Reference AET4: Tinto, Massimo, Armstrong, J. W. and Estabrook, F. B. ”Discriminating a Gravitational Wave Background from Instrumental Noise in the LISA Detector”, Phys Rev. D. 63, 021101(R) (2000). (Shows how to use zeta–insensitive to GWs but sensitive to instrumental noise–in conjunction with e.g. the Michelson combination to isolate a confusion background from instrumental noise.)

    (f) Reference AET5: Tinto, M., Estabrook, F. B. and Armstrong, J. W. ”Time-Delay Interferometry for LISA”, Phys. Rev. D, 65, 082003 (2002). (Considers the complications when the lasers do not have the same center frequencies and there are Doppler shifts due to orbital motion. Includes USO noise. TDI restated in terms of phase measurements.)

3. **Shane Larson, “Spaceborne Gravitational Wave Observatory: Sensitivity Curve Generator”; http://www.srl.caltech.edu/ shane/sensitivity/MakeCurve.html** . This is web-based software for generating noise curves for LISA and LISA-like interferometers. The noise curve is for the Armstrong-Estabrook-Tinto X configuration, in the limit of equal arms. Using this software you can choose your own parameters for a LISA-like mission (e.g. arm length, telescope diameter, laser power, ...) and generate the resulting noise curve. This software is based on semi-analytical formulae derived in:

    (a) S.L. Larson, W.A. Hiscock and R.W. Hellings, “Sensitivity for spaceborne gravitational wave interferometers,” Phys. Rev. D **62**, 062001 (2000). Available on course web site.

EXERCISES

1. **Transfer function for LISA’s GW’s.**

    (a) From the analysis of the response of LISA to gravitational waves coming in from a fixed direction and with fixed polarization, as derived in Exercise 4a of Week 15, deduce an analytic expression for the following transfer function

    $$R_X(f) \equiv S_X(f)/S_h(f) \tag{1}$$

    in the case of equal arm lengths, no orbital doppler shifts, and lasers with the same center frequencies (the case of Ref. [3a]; and of AET3, i.e. Ref. [2d], with equal arm lengths). This $R_X(f)$ is a variant of the transfer function that Armstrong and colleagues combine with their shot noise $S_X(f)$ to deduce $S_h(f)$.

    (b) In Eq. (4.1) and the preceeding paragraph of AET3 (Ref. [2d]), Estabrook, Tinto and Armstrong give an expression for $S_X(f)$. Combine it with your transfer function $R_X(f)$ to deduce $S_X(f)$.

    (c) Compare with your answer (or Yanbei Chen’s answer) to Exercise 2(a) of Week 17 [the revised version].

2. **Accuracy of knowledge of arm lengths.** In his lecture, Armstrong asserted that, in order to successfully cancel out laser frequency noise to the required accuracy in time-delay interferometry, it is necessary to know LISA’s arm lengths to an accuracy of 30 meters. Explain where this 30-meter number comes from.

3. **Effect of proof-mass acceleration noise on LISA’s α Data Channel.** This exercise, provided by Armstrong, relies on Sections III and IV of Reference AET3.

The various Time-Delay Interferometry (TDI) combinations are composed of time-shifted linear superpositions of the y_{ij} (=fractional frequency fluctuations of the signals exchanged between spacecraft) and the z_{ij} (=metrology signals exchanged between optical benches within each spacecraft). The superpositions are chosen such that the leading noise sources (the frequency fluctuations in all six lasers) are canceled. In forming the TDI-combinations to cancel the laser noise, the remaining noises (e.g. optical path noise and proof-mass acceleration noise) are modulated.

For example, the TDI-combination alpha has contributions from all six of the proof-masses. If$v_1, v_2, v_3, v_1^*, v_2^*, v_3^*$ are the scalar random processes associated with the six proof mass velocity vectors (time integrals of proof-mass acceleration vectors) dotted into the appropriate unit vector between spacecraft pairs, then the proof mass noise contribution in the α time series is [AET3, Eq. (3.4)]:

$$\begin{aligned}\alpha_{\rm PM}(t) &= v_2(t-L_3) - v_2(t-L_1-L_2) + v_3^*(t-L_2) - v_3^*(t-L_1-L_3) \\ &\quad + v_3(t-L_1-L_3) - v_3(t-L_2) + v_1^*(t) - v_1^*(t-L_1-L_2-L_3) \\ &\quad + v_1(t) - v_1(t-L_1-L_2-L_3) + v_2^*(t-L_1-L_2) - v_2^*(t-L_3)\ , \end{aligned} \tag{2}$$

where L_1, L_2, L_3 are the light travel times in the arms opposite spacecraft numbers 1, 2, 3.

(a) Show, for the special case of equal arms $L_1 = L_2 = L_3 = L$ and for the case where the six proof-mass motions (v_1, v_2, etc.) are statistically independent having individual power spectra $S_v(f)$, that the proof-mass contribution to the power spectrum of α is [AET3, Eq. (4.2)]:

$$S_\alpha^{\rm PM}(f) = [8\sin^2(3\pi f L) + 16\sin^2(\pi f L)]S_v(f)\ . \tag{3}$$

(b) In her lecture, Schumaker focuses on the proof-mass acceleration noise $S_a(f)$. What is $S_\alpha^{\rm PM}(f)$ in terms of $S_a(f)$?

4. **Different Conventions for LISA Noise Curves.**

Larson shows one version of the LISA noise curve on the web, Ref. [3]. Estabrook, Tinto and Armstrong show a different version in Fig. 3 of AET3. These two noise curves are claimed to be identical, but they look somewhat different; see especially the scales on the left axis. Derive the mathematical formula that transforms one curve into the other.

Research Problems Suggested by Schumaker.

Bonny Schumaker has suggested the following (small) research problems. She would be pleased to talk with anyone who pursues them. The TAs will not provide solutions! *If you wish, do just one of these problems for this homework set (i.e., skip all the above problems and focus in on one of these).* Schumaker can be contacted for advice at JPL: <Bonny.L.Schumaker@jpl.nasa.gov> .

Schumaker says: I'm trying to suggest problems that involve needed or useful checks on my calculations, or problems just being looked at now, so that everyone benefits – and maybe your students get a headstart on a publications or conference paper. Reference is made to Tables 1–4 of Ref. [1], excerpts of which were in my lecture slides (see pdf file). There is also an Appendix to the reference document in which all the capacitive-sensor contributions, both to PM (proof mass) direct disturbance and to the PM-SC (proof mass - spacecraft) coupling, are derived. If someone checks that, ask them not just to do the algebra, but to explain, via Landau & Lifshitz' *Electrodynamics of Continuous Media* or other fundamental physics, the explicit inclusion of the charge in the energy expression and the minus sign in the first term.

5. **Cross-coupling:**

   In the spirit of the simplified control model presented, include a control system for the PMs for one or more of the transverse axes (non-measurement axes) and assess the level of possible disturbances from cross-coupling.

6. **Local SC magnetic field gradients and interplanetary magnetic field fluctuations (disturbances A1-A4):**

   (a) Calculate the magnitude that might be expected from local (SC) sources, check with my calculations, and offer ways to reduce it that are consistent with LISA specs and limitations (e.g., size). This includes finding a better material for the PM (density and magnetic susceptibility are key – sapphire??)

   (b) Research and verify estimates I used for interplanetary magnetic fields – DC and fluctuations. Devise scheme(s) for calibrating those fluctuations or otherwise controlling or reducing (magnetic shielding?) the debilitating low-frequency effect.

7. **Thermal radiation pressure fluctuations on PM (A9):** Check the estimates I gave for the effect (and the cause – solar flux), especially that needed factor of 1/3, justify it, and devise a viable scheme for active thermal control at some level in the SC that would reduce this disturbance.

8. **Capacitive sensors:** As noted above, check my appendix calculations and the inferences that led to all the disturbances (A11-A15) and PM-SC coupling contributions (Ks1-Ks4).

9. **Patch-field effects on the electrodes:** A minor research effort would be helpful, with a clear explanation of the causes of the work-force variations and ways we might minimize them, for relevant size scales (and, by the way, what are those relevant size scales?) If they're motivated, they can call Jordan Camp at Goddard and get a head start, which might get them to give us answers that are more valuable to us than just quoting old papers.

10. **Gravitational PM actuation:** Verify my numbers and go beyond them to show how much motion of the Control Masses (CMs) (and which ones, located where, of what mass) is needed to give us the PM motion requirements I laid out on VG #45 or so – 100 micron, 5 millirad, with a DC force no larger than 100 pN. They can start by treating PM and CMs as points, but they should all be able to do the integrals to treat the PM as a finite cube. (Yes, I know it's easier to treat it as a sphere and do a multipole expansion in Y_{lm}'s, but sorry, we don't need that, and the integrals turn out to involve the same lousy logarithms). If

they solve the inverse force equations and get reasonable answers, I want to know about it! Anyone who chooses this problem may e-mail me and I’ll send them what we’ve done on it so far, with references, so that they can ”stand on our shoulders” and go further. They’ll get a conference paper, to boot.