Ph 237b: Gravitational Waves 15 May 2002

WEEK 17: LISA Overview, and LISA's Lasers and Optics

Revised version, 16 May

Lecture 31 by William Folkner (JPL) [LISA Overview]
Lecture 32,by Robert Spero (JPL) [LISA's Lasers and Optics]

Reading Related to These Lectures:

Items in bold are recommended; others are supplementary.

1. *LISA System and Technology Study Report*, European Space Agency Report SCI(2000)11, July 2000; available on our course web site and also at
   ftp://ftp.ipp-garching.mpg.de/pub/grav/lisa/sts/sts_1.05.pdf .
   (a) **Chapter 3, The LISA Concept – An Overview.**
   (b) **Chapter 4, Measurement Sensitivity.**
   (c) **Chapter 5, The Interferometer.**
2. Other, earlier studies of the LISA Mission, and a popular-level LISA brochure, available on JPL's LISA web site at http://lisa.jpl.nasa.gov/documents.html .
3. *Proceedings of the First International LISA Symposium*, *Classical and Quantum Gravity*, **14**, No. 6 (June 1997). Available on the web at http://www.iop.org/EJ/S/1/NCA143559/
4. *Proceedings of the Second International LISA Symposium*, AIP Conference Proceedings Volume **456**, edited by W.M. Folkner (American Institute of Physics, Woodbury New York, 1998).
5. *Proceedings of the Third International LISA Symposium*, *Classical and Quantum Gravity*, **18**, No. 19 (7 October 2001). Available on the web at http://www.iop.org/EJ/S/1/NCA143559/

EXERCISES

I have added a new Exercise 2 and bumped subsequent exercise numbers up by one; and I have added some new remarks in Exercise 3a – Kip.

1. **Dependence of Shot Noise on LISA's Design Parameters**

   Denote by $P_{\rm out} \simeq 1$ W the power in the outgoing laser laser beam from one of LISA's spacecraft, by $r_t \simeq 15$ cm the radius of LISA's telescope mirrors, by $L \simeq 5 \times 10^6$ km LISA's arm length, by $\lambda_e \simeq 1\mu$m the wavelength of LISA's light, and by f the gravitational-wave frequency. Express all answers to the following questions in terms of these parameters together with fundamental constants such as the speed of light and Planck's constant. For simplicity confine attention to gravitational waves with wavelength longer than the separation L between spacecraft.

(a) What is the frequency range f to which we are confining attention?

(b) What is the mean light power ("signal power") P_s received in the telescopes of the receiving spacecraft? What is the spectral density $S_{P_s}(f)$ of the shot noise in this received power?

(c) Assume that the receiving optics and electronics are so designed that the accuracy with which the phase $\phi(t)$ of the incoming light can be measured is constrained only by this shot noise and by other noises that have a total noise power no greater than the shot noise. (Spero's lecture focused on how this can be done.) Explain why this means that $S_\phi(f) \sim S_{P_s}(f)/P_s^2$. Evaluate $\sqrt{S_\phi(f)}$ in terms of LISA's parameters.

(d) What is the corresponding noise in the gravitational-wave measurement? Your answer, aside from a factor of order unity, should be

$$\sqrt{S_h(f)} \sim \sqrt{\frac{h_{\rm Planck} c/\lambda_e}{P_{\rm out}}} \frac{\lambda_e^2}{2\pi^2 r_t^2} \; .$$

Explain physically why this noise is independent of LISA's arm length L.

(e) Insert numbers into your formula for $\sqrt{S_h(f)}$ and compare with the "official" LISA noise curve. (You will have to convert to the units and conventions of whatever official noise curve you use.)

(f) It might become necessary to reduce this noise by a factor 4 in order to ensure LISA's ability to detect waves from the inspiral of compact bodies into supermassive black holes. If this becomes necessary, what changes in parameters do you think might be the most practical and least expensive? (This is a question that the LISA International Science Team has asked the LISA Project to look at.)

2. **NEW: LISA Shot Noise for GW's with Arbitrary Wavelength**

In the previous exercise, you restricted yourself to GW wavelenths longer than the earth-spacecraft separation. Now consider gravitational waves with arbitrary wavelength, coming from an arbitrary direction. Adopt the geometry of Exercise 4 of Week 15: LISA's corner drag-free spacecraft is at the spatial origin, one end spacecraft is at $x = \alpha L$, $y = \beta L$, $z = \gamma L$, and another end spacecraft at $x = \alpha' L$, $y = \beta' L$, $z = \gamma' L$. Laser light is sent out from the corner spacecraft to the two end spacecraft which transpond the light back phase coherently, and the two returning light beams are then interfered so as to deduce their difference in phase. The gravitational wave travels in the z direction and has $+$ polarization.

(a) By combining the result of Exercise 4(a) of Week 15 [cf. page 10 of Yanbei Chen's solutions, on the course website] with the result of Exercise 1(c) above, derive the shot-noise spectral density $S_h(f)$ for waves propagating in the chosen direction and with the chosen polarization.

(b) Plot this spectral density for the special case where the detector lies in the plane orthogonal to the wave's propagation ($\gamma = \gamma' = 0$). Explain the physical origin of the infinite peaks in the noise.

(c) Plot your spectral density for a more random orientation of LISA: ($\alpha = \sqrt{19/100}, \beta = 0, \gamma = 9/10$); ($\alpha' = 0, \beta' = \sqrt{56/81}, \gamma' = 5/9$). Explain why the infinite peaks of noise that you saw in your previous plot are smeared out, and explain why the number of oscillations is larger.

(d) It is conventional to plot the LISA noise curve inverse averaged over the sky and over linear polarizations,

$$\frac{1}{S_h^{\rm SA}(f)} \equiv \left\langle \frac{1}{S_h(f)} \right\rangle . \tag{1}$$

Here $\langle ... \rangle$ is the average, performed holding LISA's orientation fixed and allowing the direction to the source and the polarization to vary randomly over all possibilities. Explain why this is a reasonable way to do the averaging — i.e., why not define $S_h^{\rm SA}(f) \equiv \langle S_h(f) \rangle$? Explain why the averaging produces the kinds of mild oscillations in the noise curve that are shown in the "official" noise plots — e.g., slide 9 of Folkner's lecture.

(e) By combining the results of this exercise and the previous one, and by knowing that the rise of the LISA noise curve at low frequencies is due to stochastic forces that act on the LISA proof mass (to be studied next week), explain why changing the LISA arm length causes the sky averaged LISA noise curve to change in the manner shown in Folkner's slide 11. Among other things, explain why, when the arms are shortened by a factor 5, (i) the oscillations move rightward by the amount shown, (ii) the rising (stochastic-force) left part of the noise curve moves right by the lesser amount shown, (iii) the floor of the noise curve broadens, and (iv), the floor does not change its height.

The following exercises, devised by Bob Spero and edited by Kip, are designed to help you understand more deeply the material in Spero's Lecture 32. You are strongly encouraged to study the video of Spero's lecture while working these exercises. By going back and forth between his pdf slides, the video, and the exercises, you may learn a lot.

3. **LISA with Direct Detection of the Laser Light**

Slide 4 of Spero's Lecture 32 shows the Noise-Equivalent-Power, NEP, of several commercial photodiodes. (Recall from Spero's lecture that "NEP" is the power that can be detected at signal to noise ratio unity.) The NEP is plotted against "photodetector bandwidth" f— a quantity that has a different meaning than the bandwidths we normally deal with: it is the rate at which the photodetector is required to respond to changes of light intensity, i.e. the frequency of the light's intensity changes. By contrast, the "$1/\sqrt{\rm Hz}$" in the units of the NEP refers to the usual bandwidth of the measurement — e.g., when one is doing a long-term averaging for a time $\hat\tau$, the usual bandwidth is $\Delta f = 1/\hat\tau$; and one should multiply NEP by the square root of this bandwidth to get the oscillating power that is detectable at frequency ("photodetector bandwidth") f.

(a) Suppose that the arm lengths in LISA were constant to within a small fraction of a wavelength instead of changing rapidly, and our goal was to monitor tiny changes of relative armlength in the LISA frequency band — as is the case in LIGO. What would be the Noise-Equivalent Displacement, "NED", i.e., the relative displacement "per root Hertz" that would be detectable at signal to noise ratio unity using these photodiodes and one-watt lasers? **NEW:** What would be the necessary photodetector bandwidth in this arrangement, and what would be the contribution of shot noise to LISA's $S_h(f)$. [Your answer should be a far larger noise than Exercises 1 and 2 claim is achievable. This is a warning that, even if LISA's arm lengths were not changing rapidly, direct detection of the incoming photons would be a bad experimental strategy.]

(b) In LISA the arm lengths are changing at a rate of several meters per second. What photodetector bandwidth is needed in order to monitor this arm length motion? (Recall Spero's remark in his lecture, that the meaning of bandwidth for a photodetector is the speed with which the detector can respond to changes in the intensity of the light.) What would be LISA's gravitational-wave sensitivity if it used direct detection and one of these photodetectors?

4. **LISA with Offset Locking to Monitor the Phase of the Incoming Laser Signal**

The previous exercise demonstrated the hopelessness of direct detection as a technique for monitoring the incoming laser signal. A promising alternative technique is "Offset locking" of a laser ("local oscillator") on the receiving spacecraft. Offset locking is discussed in Spero's slide 5. Here you will explore some details.

One key principle that underlies offset locking is the interference (beating) of the incoming laser light and light from the local oscillator. This interference is measured by combining the light beams in a 50/50 beam splitter and monitoring the intensity of the combined beams with a photodetector like those discussed in the previous exercise; see the diagram in Spero's slide 5. Suppose the two light beams have the same frequency but differ in phase by ϕ (which may depend on time). Denote by P_s the mean power in the incoming signal light and by P_L that in the local-oscillator light, and by $R \equiv P_L/P_s$ their ratio. To enable us to detect and monitor the signal in the presence of the photodetector's noise, we must make the intensity of the light falling on the photodiode be sufficiently high as to overcome the photodiode's own noise. This requires $R \gg 1$.

(a) What is the peak-to-peak intensity variation, as measured by the photodetector, as the phase ϕ changes by 2π?

(b) What value of R is required for the shot noise of the incoming signal to equal the photodetector's noise? What is the corresponding required local oscillator power P_L?

(c) In the optical-electronic configuration shown in Spero's slide 5, the local oscillator (laser in the receiving spacecraft) is locked to the input light, plus an offset frequency of typically 10 MHz. The laser can be tuned by changing the length of its optical resonator with a piezo-actuated mirror. Estimate the maximum tuning range of such a laser, as limited by mode spacing in the optical resonator.

(d) Owing to finite gain of the control system, the detected signal will contain some phase variation from the local oscillator in addition to that of the signal. Estimate the maximum suppression factor of this noise as a function of frequency, using one of the photodiode/amplifiers from slide 4.

5. **Acoustic-Optic Modulator, and Offset-Cancelled Locking of a Transponded Light Beam to an Incoming Signal Beam.** Spero's slide 6 (bottom diagram) introduces the AOM, or acousto-optic modulator. Information about these devices is available in textbooks and on the web. You may be able to figure out how they work from first principles and work this exercise without looking at textbooks or the web; but if you take that approach, then it would be good to look at textbooks or the web afterwards, to make sure you have understood properly.

   (a) A key role of the acoustic-optic modulator is to produce two outgoing light beams from one incoming beam. This is achieved by setting up standing acoustic waves in the modulator, which produce standing waves of oscillation of the refractive index. What is the angle between the two outgoing beams as a function of the acoustic wave's frequency? Give both a formula and numbers, assuming physically reasonable values for the relevant parameters. The figure shows the undiffracted and diffracted beams striking separate mirrors; estimate the minimum required acoustic frequency to achieve this separation. (You will need to estimate the mirror sizes and the distances propagated by the light beams).

   (b) The bottom diagram in Spero's slide 6 shows a scheme called "Offset Cancelled Lock" (discussed in Spero's video) that produces a *transponded* output beam with the same frequency as the incoming signal beam. A necessary element of this scheme is the AOM, which splits the local-oscillator beam in two; and a clever feature of the scheme is its ability to compensate for fluctuations in the thermal expansion of the AOM glass. This thermal-expansion noise, before compensation, can be quantified by the fluctuating phase shift put onto both light beams, or equally well by fluctuating "displacement noise" $x_A(t)$ [defined by (phase fluctuation) = kx_A where k is the light's wave number], put onto the light. Denote by $x_A(f) \equiv \sqrt{S_{x_A}(f)}$ the square root of the spectral density of this thermally-induced displacement noise. What is $x_A(f)$ for an AOM made of ordinary glass, assuming the temperature fluctuations are $1\mu\mathrm{K}/\sqrt{\mathrm{Hz}}$?

   (c) Explain in your own words how the scheme shown in slide 6 compensates for this displacement noise, i.e. how it enables the transponded light to be free of the displacement noise.

   (d) Identify factors that might make this compensation imperfect.

6. **Frequency Offset in the Laser Readout and Transponder System.** The bottom diagram in Spero's slide 11 combines some of the ideas described above into a scheme for both reading out the phase difference $\phi_{21} = \phi_2 - \phi_1$ between an incoming beam and a local oscillator, and for producing transponded light.

   (a) Explain in your own words the reason for the frequency offset Δf that appears in this scheme.

(b) What considerations go into the choice of the value of Δf?

7. **Noise due to Time Quantization in the Phase Meter.** The phase meter that appears as a black box in Spero's slide 11 is explained in slide 13. Equation (19) of that slide is a formula for the noise due to time quantization, in the output of the phase meter. Estimate the numerical values of the parameters in this equation and the numerical values of the resulting noise in the gravitational-wave measurement, $S_h(f)$.