Prospects for gravitational wave detection with forthcoming Pulsar Timing Arrays

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OUTLINE

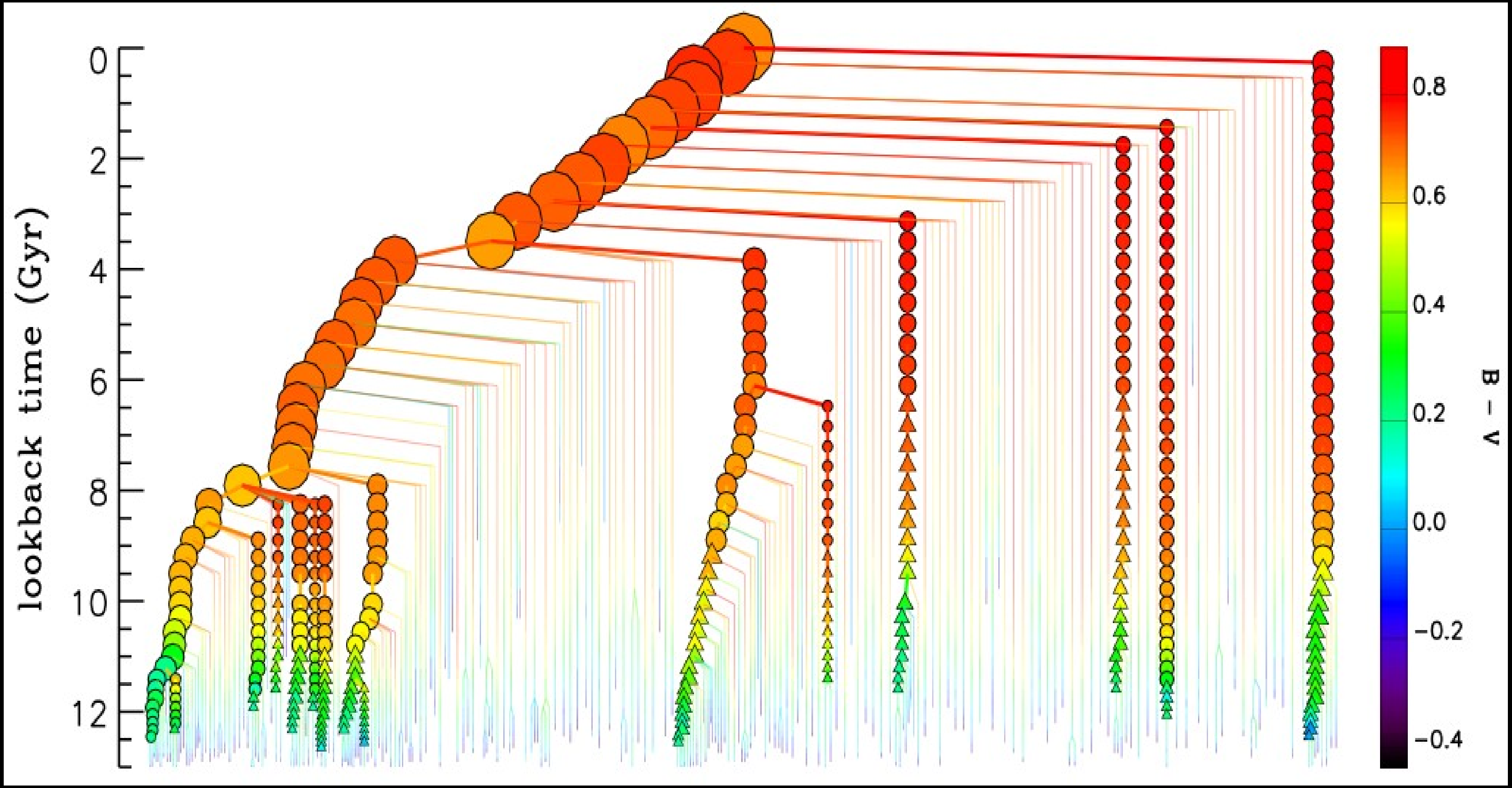
- > PTAs and MBHBs

- > GWs and PTAs: unresolved background

- > GWs and PTAs: resolvable sources

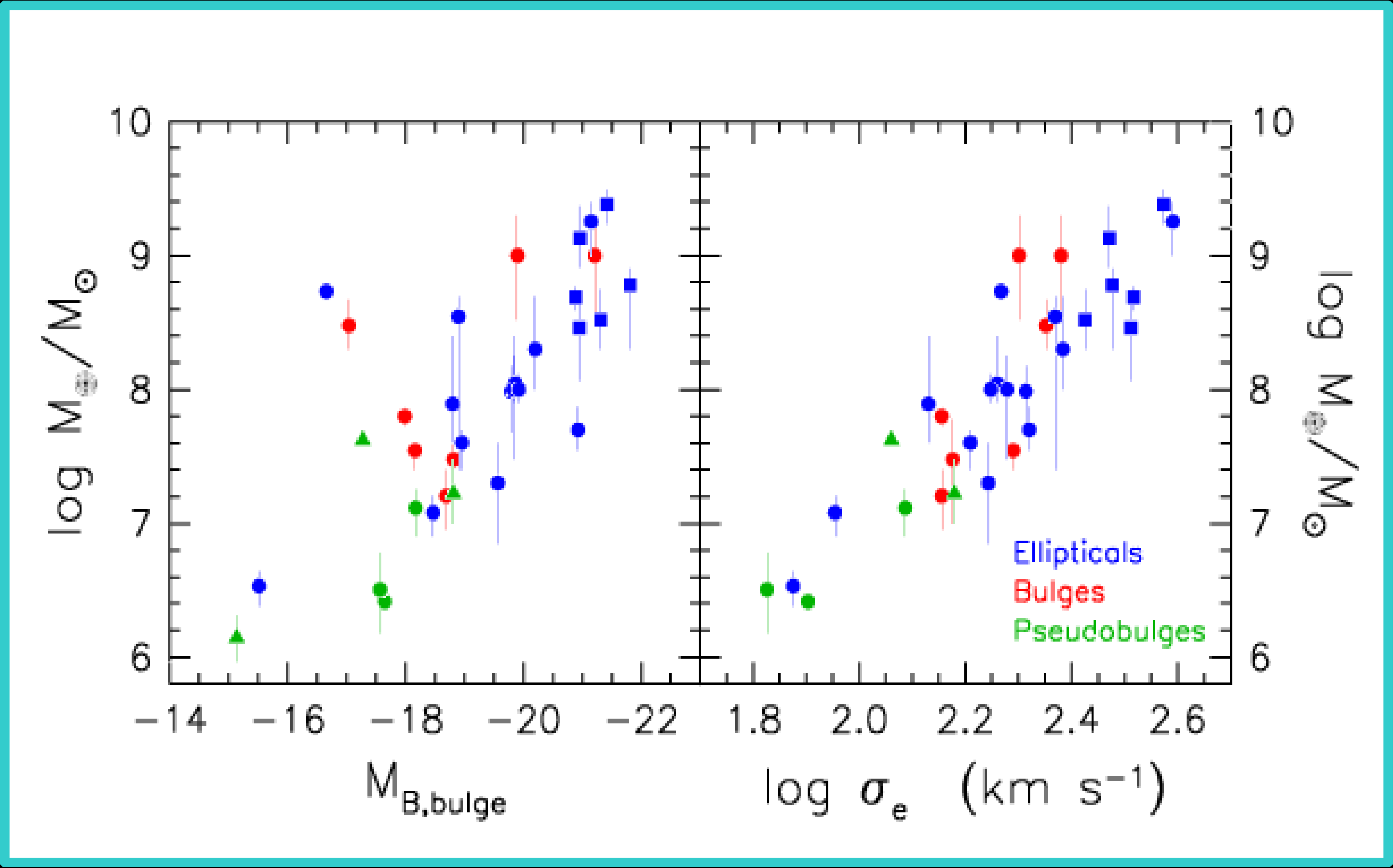
- > Parameter estimation: preliminary results

HIERARCHICAL GALAXY FORMATION…



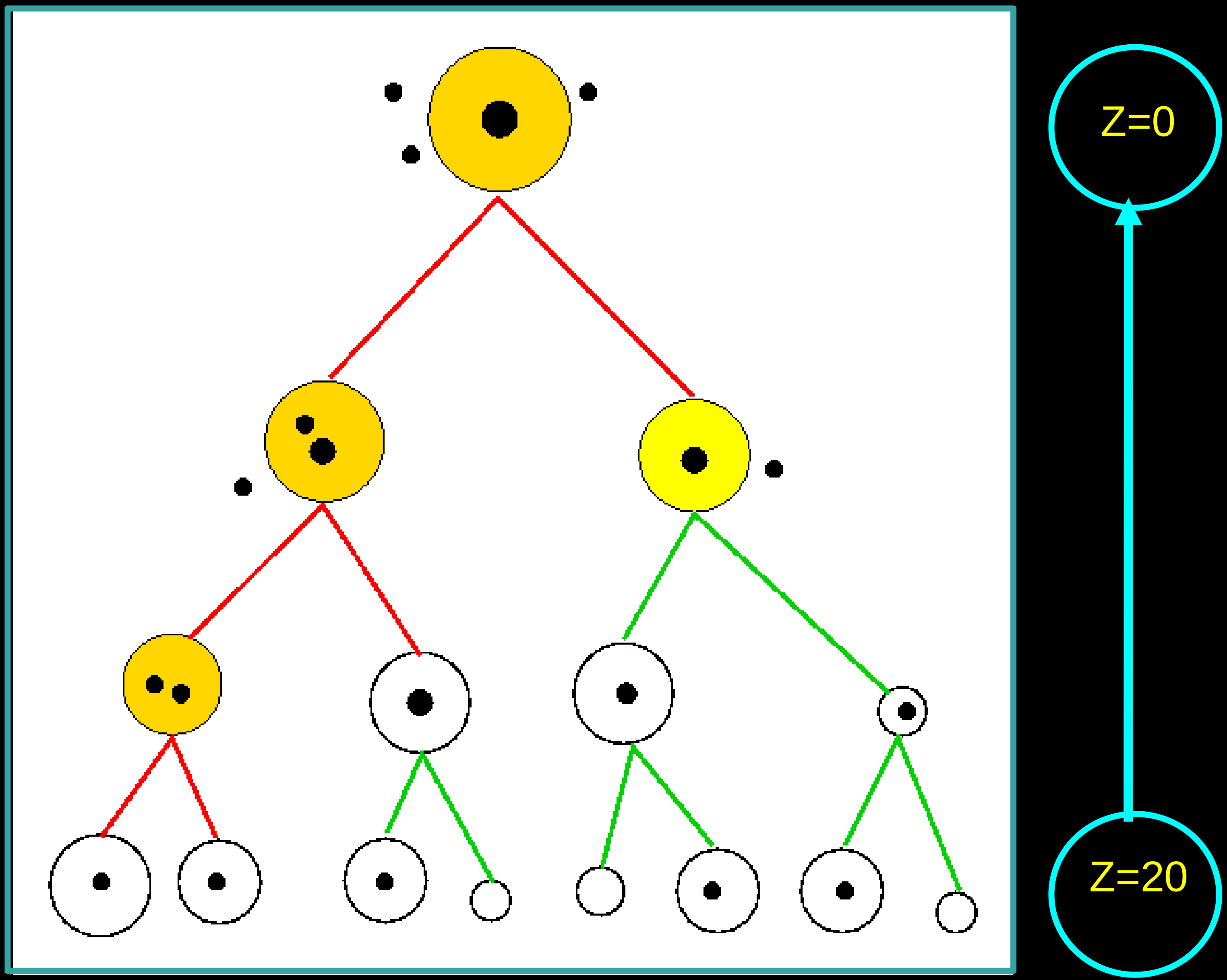
From De Lucia et al. 2006

...M_{BH}- bulge relations: co-evolution of MBHs and galaxies...

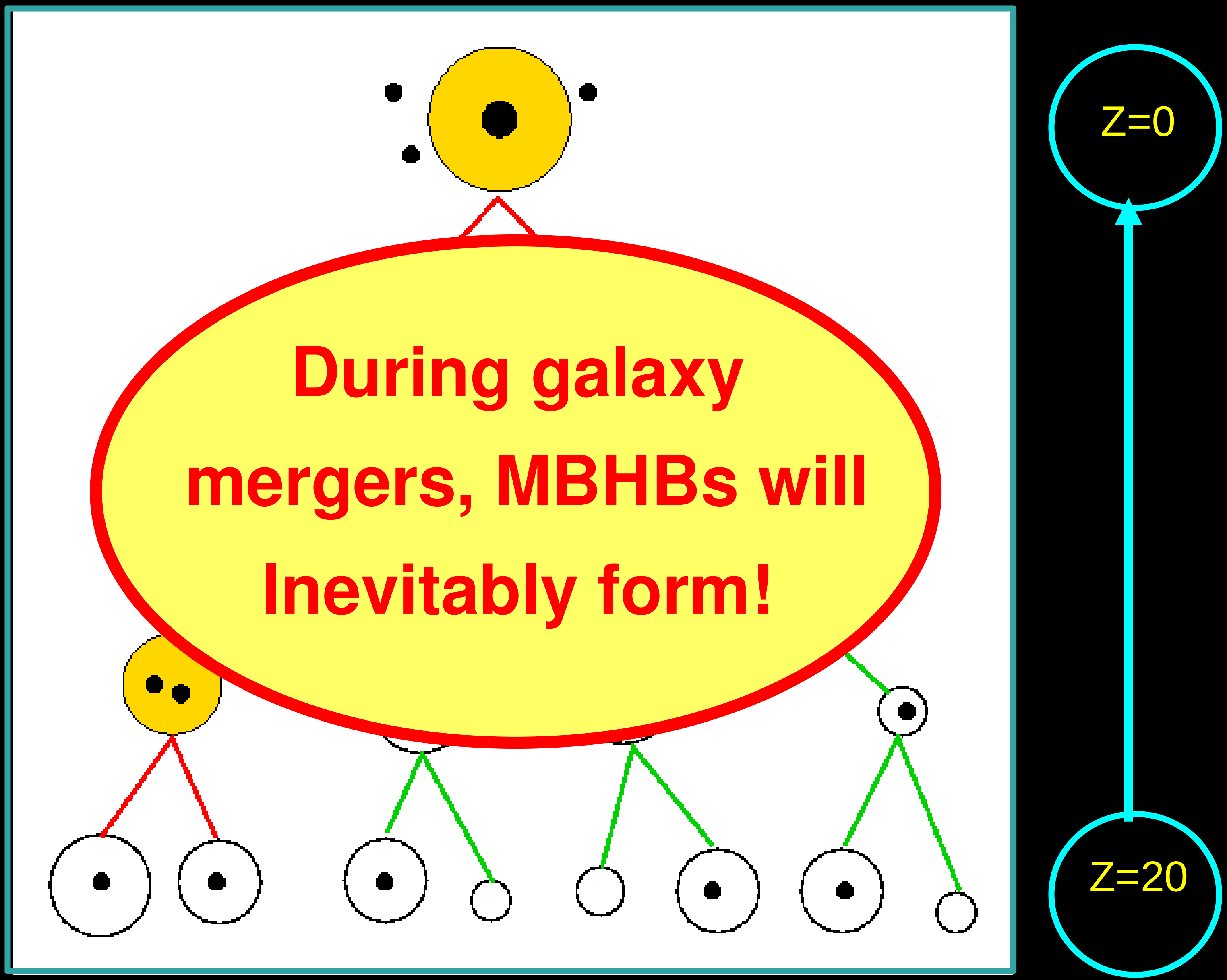


Gebhardt et al. 2000, Ferrarese & Merritt 2000, Tremaine et al. 2002...

...HIERARCHICAL MBH FORMATION



...HIERARCHICAL MBH FORMATION



SMBHs DYNAMICS

1. **dynamical friction** (Lacey & Cole 1993, Colpi et al. 2000)

- from the interaction between the DM halos to the formation of the BH binary
- determined by the global distribution of matter
- efficient only for ***major mergers*** against mass stripping

2. **binary hardening** (Quinlan 1996, Miloslavljevic & Merritt 2001, Sesana et al. 2007)

- ***3 bodies interactions*** between the binary and the surrounding stars
- the binding energy of the BHs is larger than the thermal energy of the stars
- the SMBHs create a ***stellar density core ejecting the background stars***

3. **emission of gravitational waves** (Peters 1964)

- takes over at subparsec scales
- leads the binary to coalescence

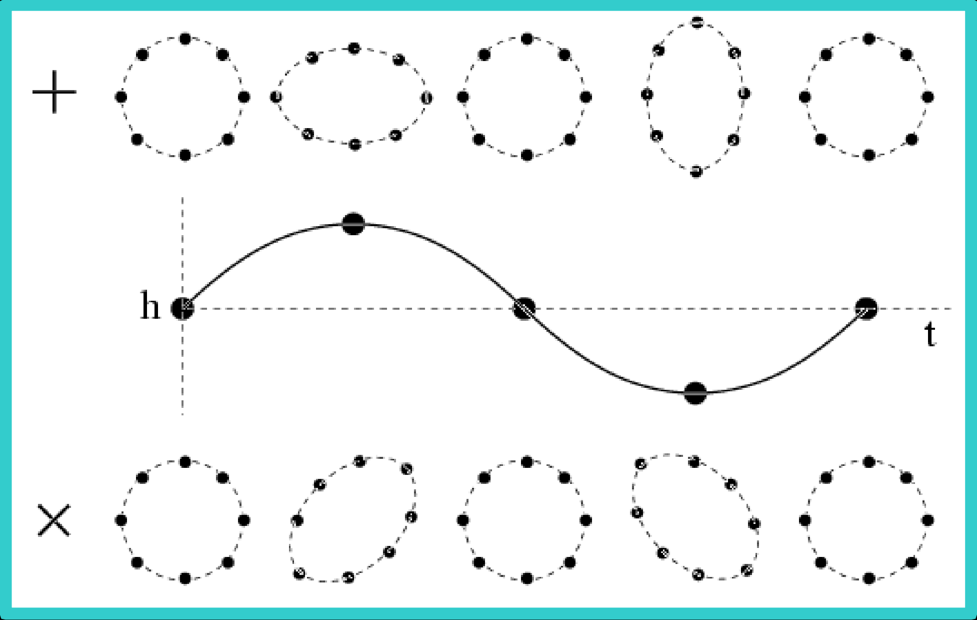
Directly from general relativity

Every accelerating mass distribution with non-zero quadrupole momentum emits GWs!

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} \ll 1$$

Perturbed Minkowski metric tensor :

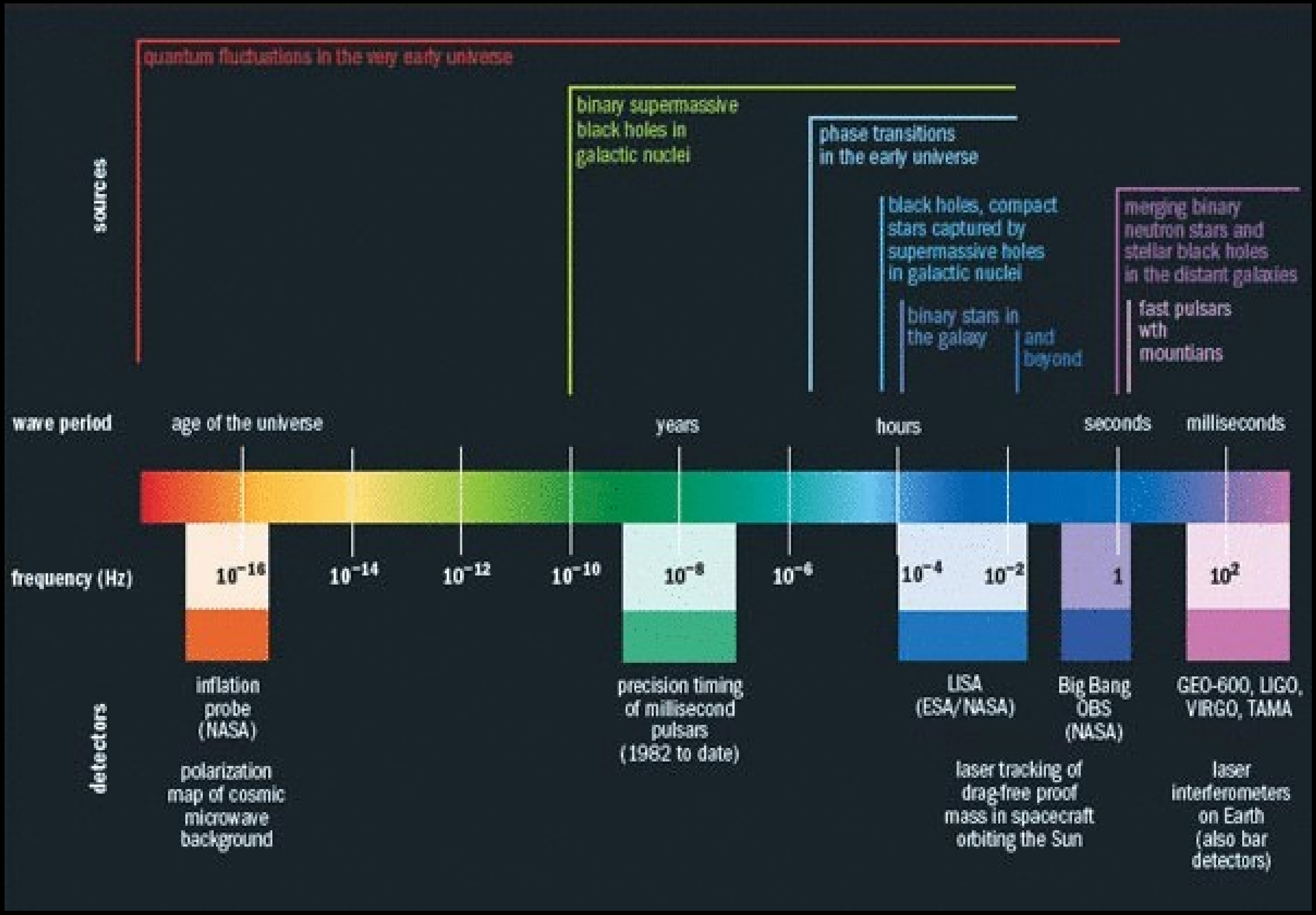
$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + h_{+}^{TT} & h_{\times}^{TT} \\ 0 & 0 & h_{\times}^{TT} & 1 - h_{+}^{TT} \end{pmatrix}$$



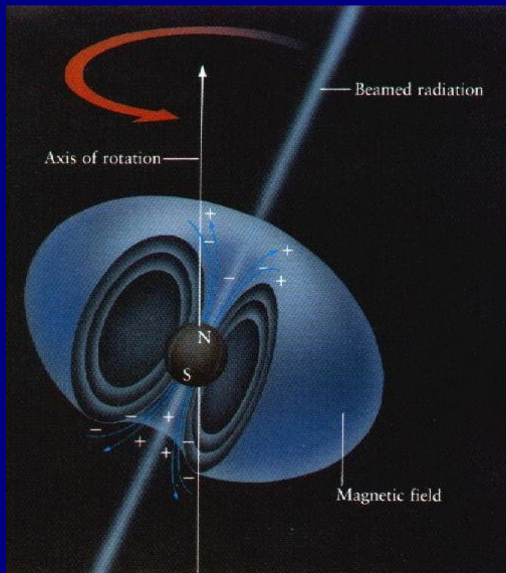
Perturbation perpendicular to the wave propagation direction

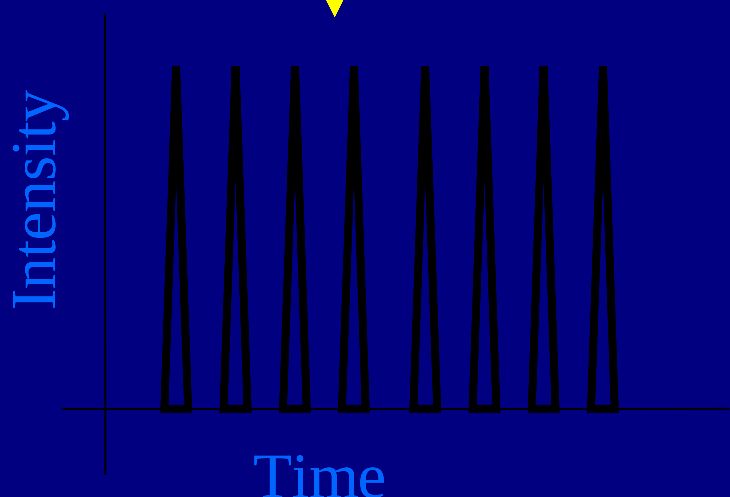
A MBHB is a perfect candidate for GW emission!

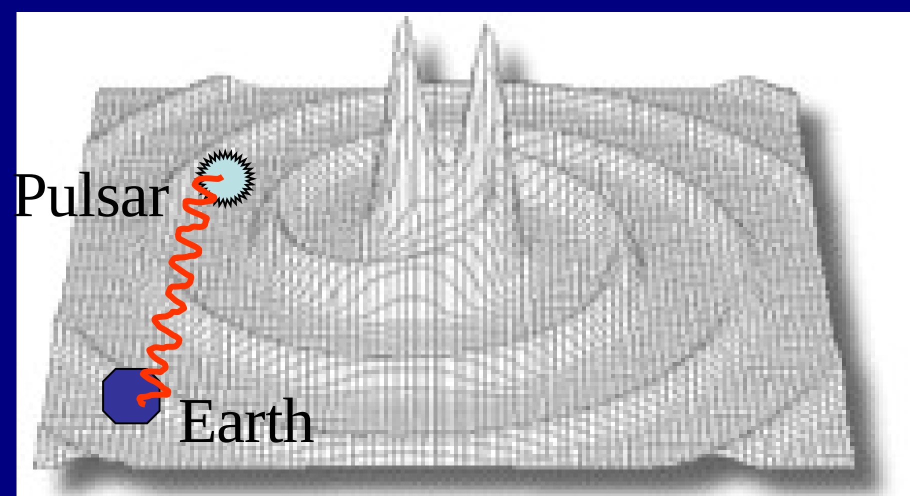
THE GRAVITATIONAL WAVE SPECTRUM



Pulsar Timing Arrays







PPTA (Parkes pulsar timing array)

NanoGrav (north American nHz observatory for gravitational waves)

LEAP (large European array for pulsars)

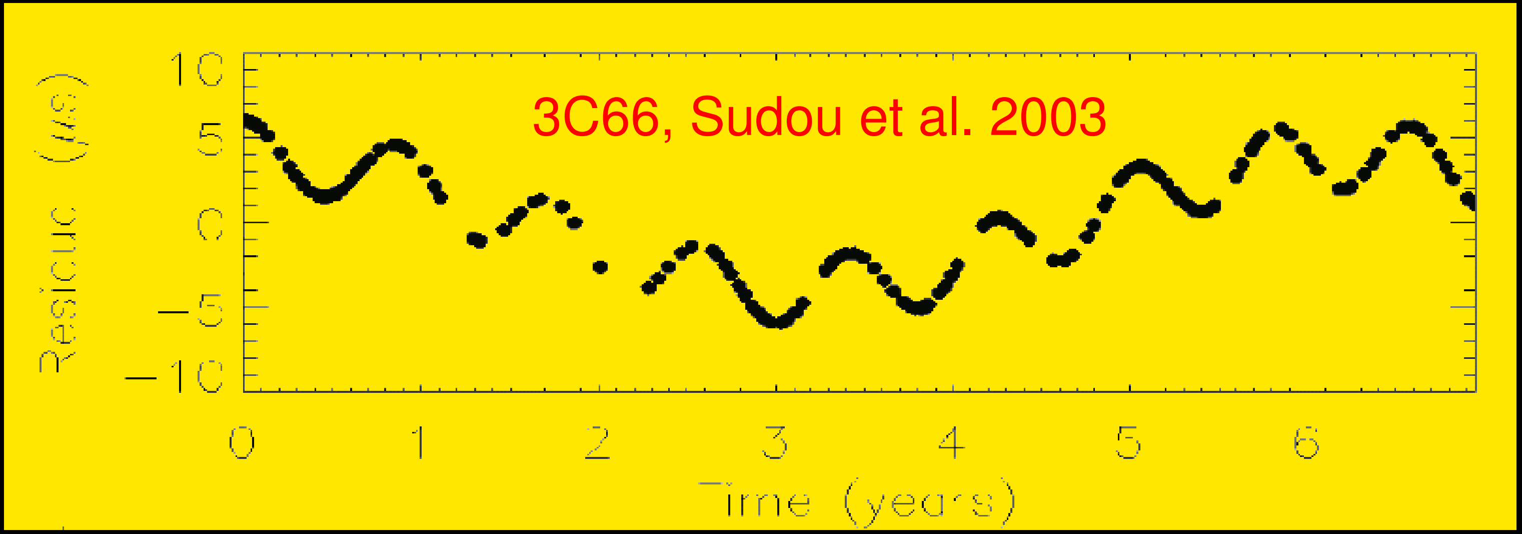
The GW passage cause a modulation of the MSP frequency

$$\frac{\nu(t)-\nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv h_{ab}(t_{\mathrm{p}},\hat{\Omega}) - h_{ab}(t_{\mathrm{ssb}},\hat{\Omega})$$

The ***residual*** in the time of arrival of the pulse is the integral of the frequency modulation over time

$$R(t) = \int_0^T \frac{\nu(t)-\nu_0}{\nu_0}\,dt$$

R~h/(2πf)



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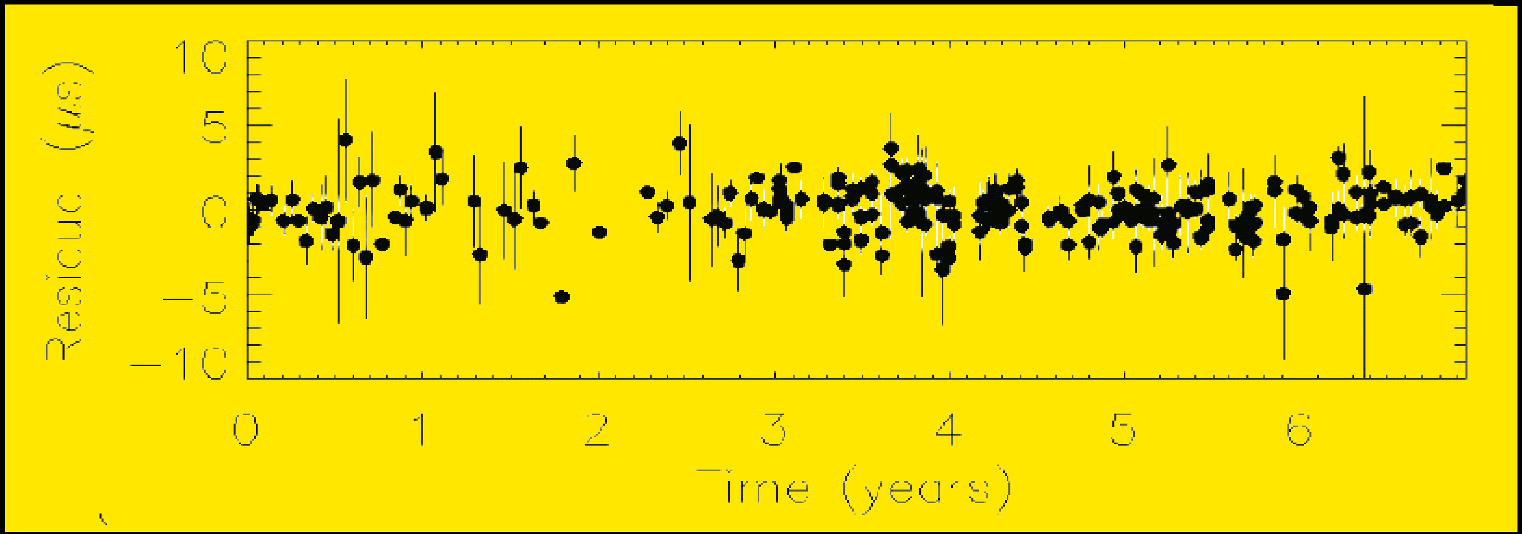
$$R(t) = \int_0^T \frac{\nu(t)-\nu_0}{\nu_0}\,dt$$

R~h/(2πf)

(Jenet et al. 2004)



Theory of GW background from SMBHs

Consider a class of sources with differential number density $d^2n/dzdM$ emitting an energy spectrum $dE/d\ln f$

$$h_c^2(f) = \frac{4G}{\pi c^2 f^2} \int_0^\infty dz \int_0^\infty d\mathcal{M} \, \frac{d^2 n}{dz d\mathcal{M}} \frac{1}{1+z} \frac{dE_{\rm gw}}{d\ln f_r}$$

$$\frac{d^2 n}{dz d\mathcal{M}} = \frac{d^3 N}{dz d\mathcal{M} d\ln f_r} \frac{d\ln f_r}{dt_r} \frac{dt_r}{dz} \frac{dz}{dV_c}$$

$$\frac{dE}{d\ln f_r} = \frac{1}{1+z} \frac{dt_r}{d\ln f_r} \frac{\pi c^3}{4G} 4\pi r(z)^2 f_r^2 h^2$$

$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M} \, \frac{d^3 N}{dz d\mathcal{M} d\ln f_r} h^2(f_r)$$

$$\delta t_{\rm bkg}(f) \approx h_c(f)/(2\pi f)$$

For MBHBs $dN/d\ln f \propto f^{-8/3}$

$$h_c(f) = A \left(\frac{f}{\mathrm{yr}^{-1}} \right)^{-2/3}$$

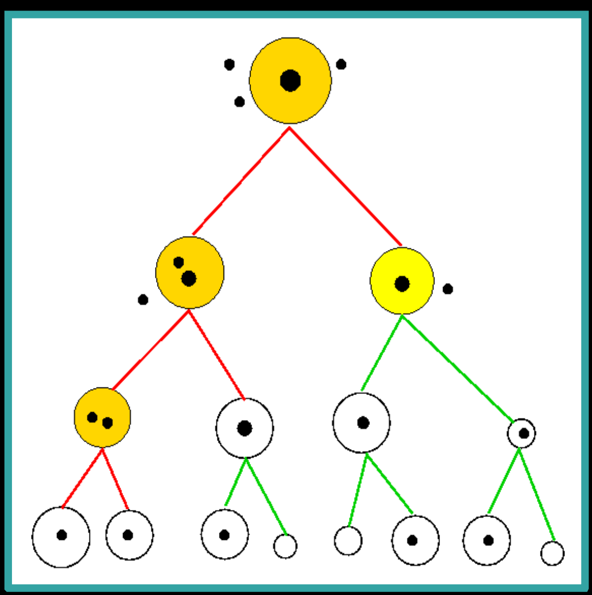
Phinney 2001, Jaffe & Backer 2003, Wyithe & Loeb 2003, Sesana et al. 2004, Enoki et al. 2004

Modelling the SMBH population

SEMI-ANALYTICAL MERGER TREES (Volonteri Haardt & Madau 2003)**:**

> *We follow backwards the merger hierarchy by means of EPS Monte-Carlo merger tree*

> *The semi-analytic code follows the **accretion** and the **dynamical history** of BHs in every single branch of the tree*

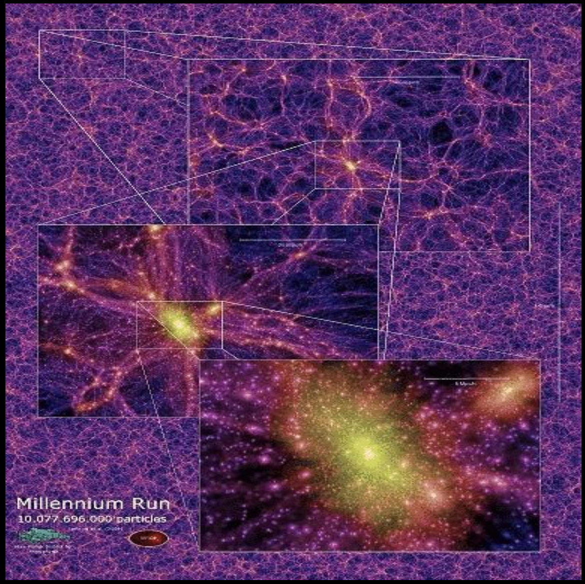


MILLENNIUM RUN (Springel et al 2005)**:**

> *N-body numerical simulations of the halo hierarchy*

> *Semi-analytical models for galaxy formation and evolution*

> *We extract catalogues of merging galaxies and we populate them with sensible MBH prescriptions*



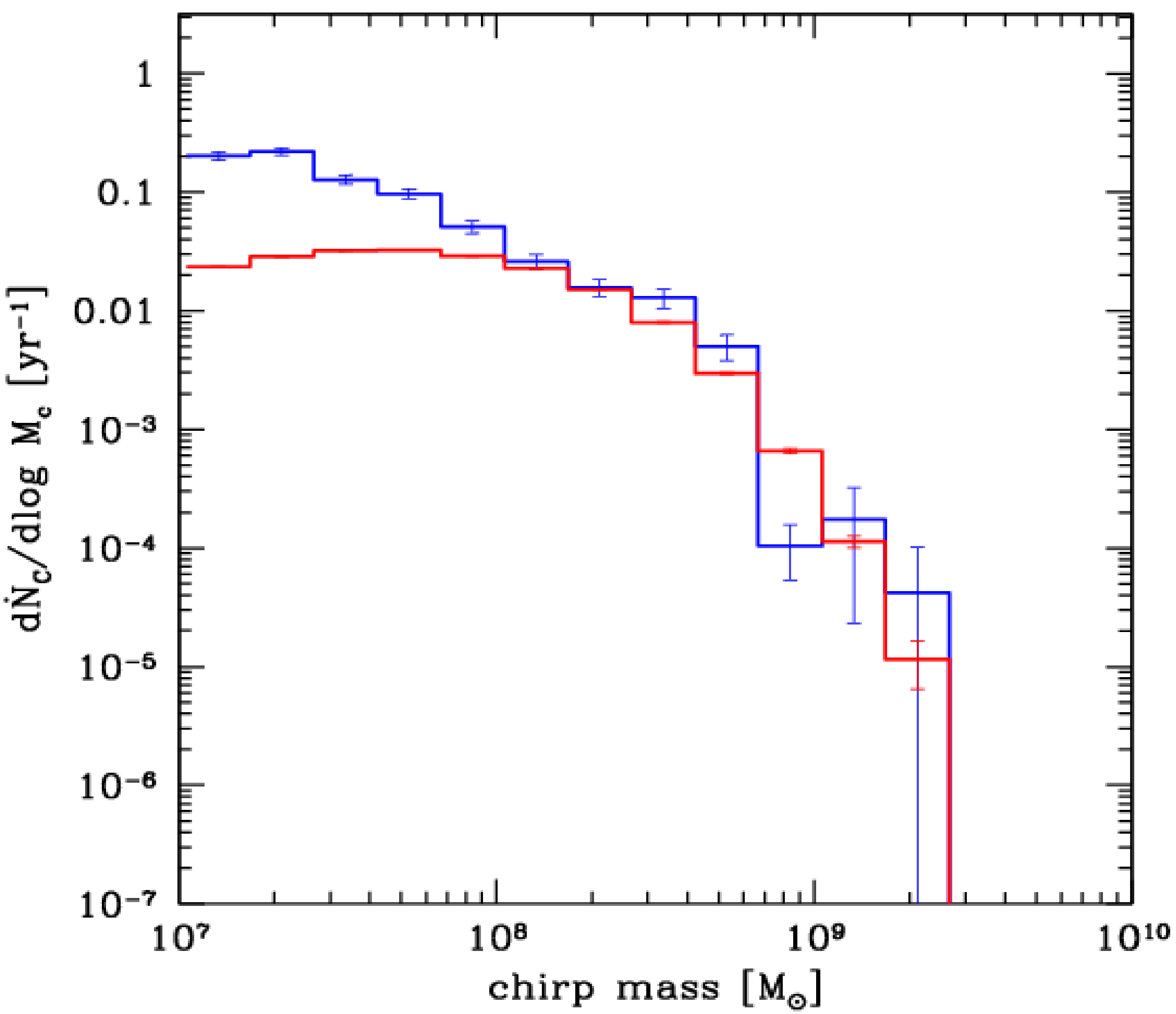
Populating bulges with MBHs

We consider several BH-host relations:

1. **M_{BH}-sigma** (Tremaine et al. 2002)
2. **M_{BH}-M_{bulge}** (Tundo et al. 2007)
3. **M_{BH}-M_{bulge} z dependent** (Mclure et al. 2006)
4. **M_{BH}-L_{bulge}** (Lauer et al. 2007)

For any relation we employ three different accretion prescriptions:

a- **Accretion after merger**
b- **Accretion only onto M_1, before merger**
c- **Accretion on both MBHs before merger**

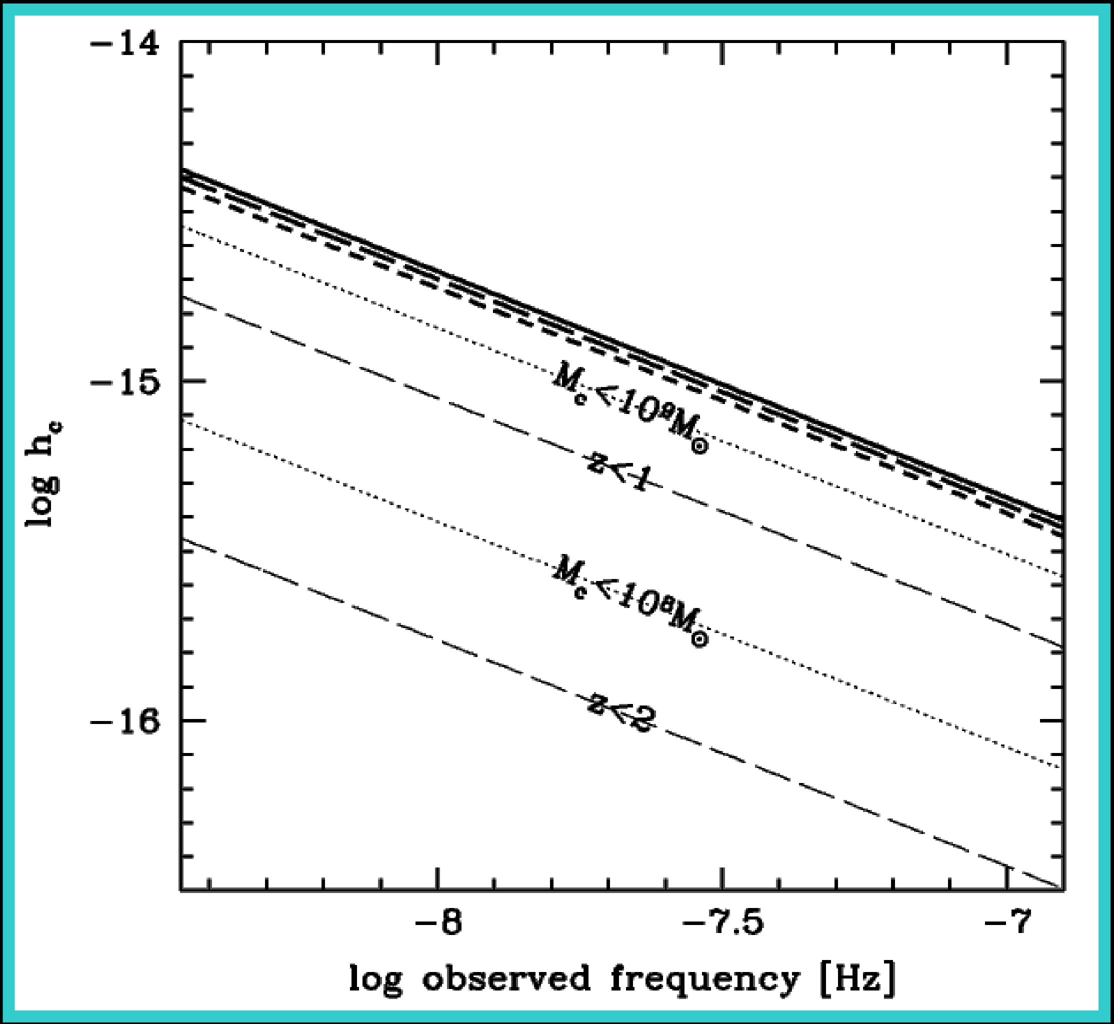


1
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$d\dot{N}_c/d\log M_c$ [yr^{-1}]
10^7
10^8
10^9
10^{10}
chirp mass [$M_\odot$]

Theoretical background

>Sensitivity in the frequency range ***10^{-9}-10^{-7} Hz***

>Signal dominated by MBHBs with **masses *>10^{8} $M_{\odot}$ at z<2***

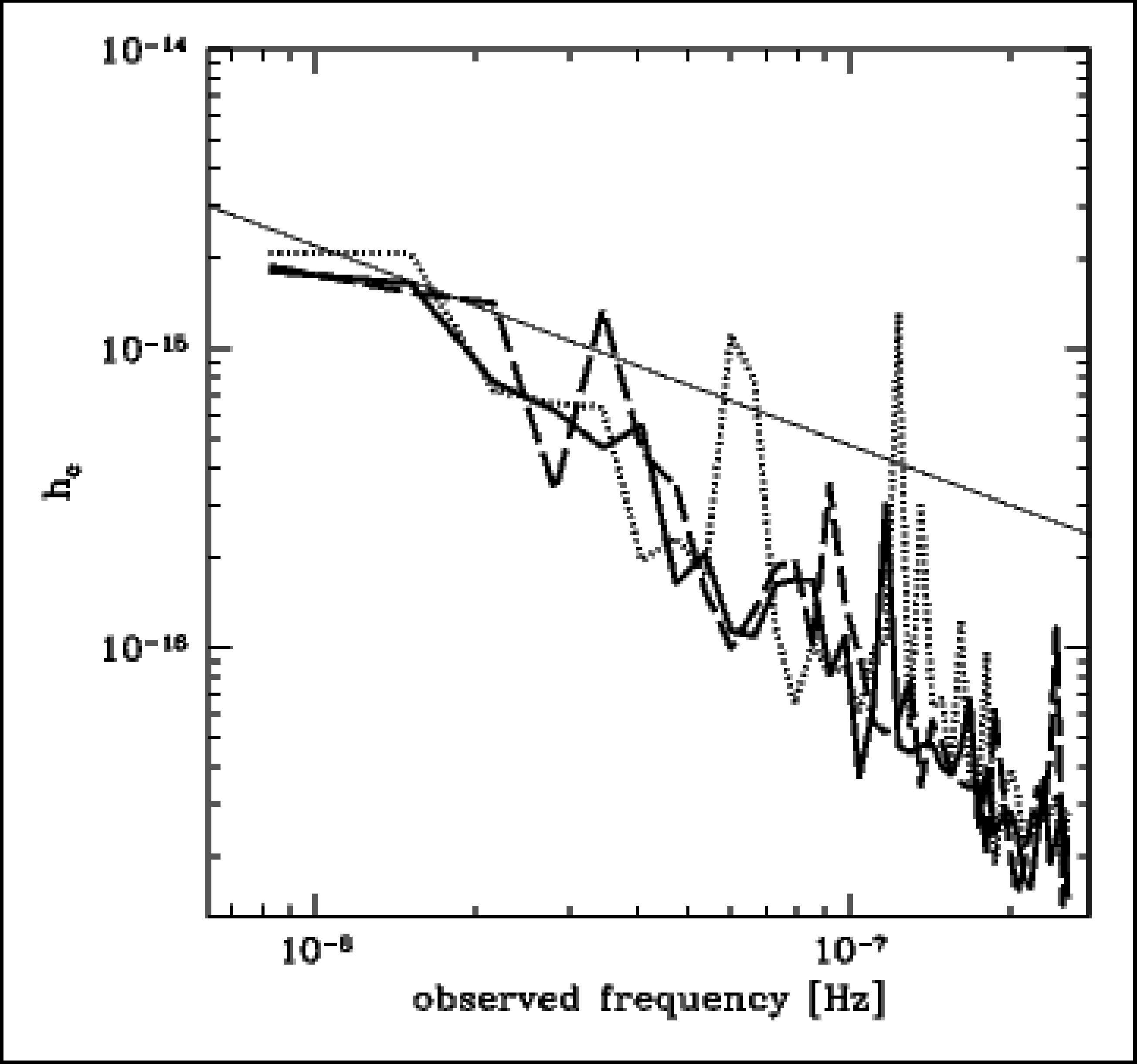


>Probe the MBHB merger rate at low redshift

>Constrain the local MBH mass function?

>Is insensitive to the seed BH population

Simulated Montecarlo signal



-is not smooth at all

-significantly deviate from the -2/3 slope

A simple sampling issue

Let us consider the probability density function of sources per unit chirp mass $p(\mathcal{M})$, so that

$$\int_0^\infty p(\mathcal{M})\, d\mathcal{M} = 1\,. \qquad (15)$$

We sample now $p(\mathcal{M})$ with N objects (sources). The total number of objects N is model dependent, but we keep it general for the time being; we can define $\bar{\mathcal{M}}(N)$ such that

$$\int_{\bar{\mathcal{M}}}^\infty p(\mathcal{M})\, d\mathcal{M} = \frac{1}{N}\,. \qquad (16)$$

For any function h – this will be the GW strain – we can always define its value weighted by the probability distribution $p(\mathcal{M})$ as

$$\bar{h}^2 = \int_0^\infty p(\mathcal{M}) h^2(\mathcal{M})\, d\mathcal{M}\,. \qquad (17)$$

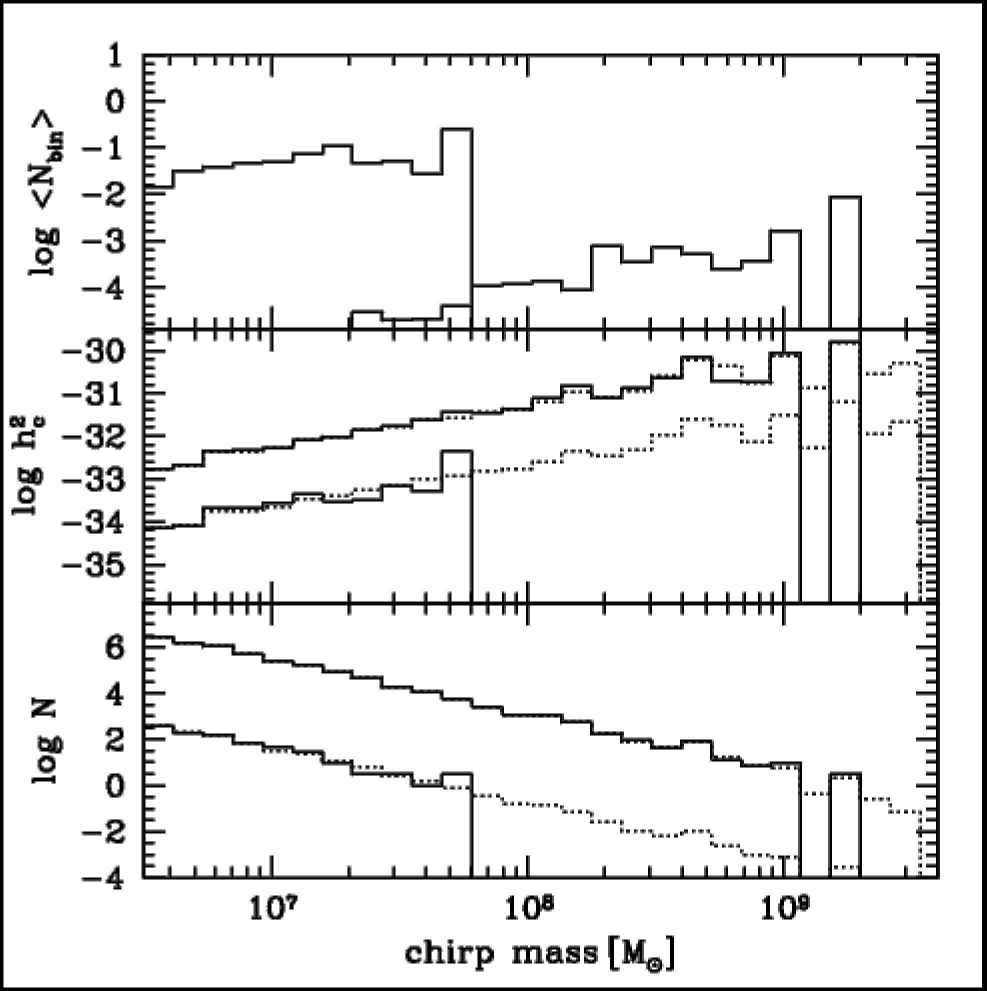
For a fixed value of N, we define the following quantity:

$$Z \equiv \frac{\int_{\bar{\mathcal{M}}}^\infty p(\mathcal{M}) h^2(\mathcal{M})\, d\mathcal{M}}{\int_0^\infty p(\mathcal{M}) h^2(\mathcal{M})\, d\mathcal{M}}\,; \qquad (18)$$

Z is the fraction of h^2 whose contribution comes from less than one source, and is therefore not actually present. The overall value of h sampled with N objects is then simply

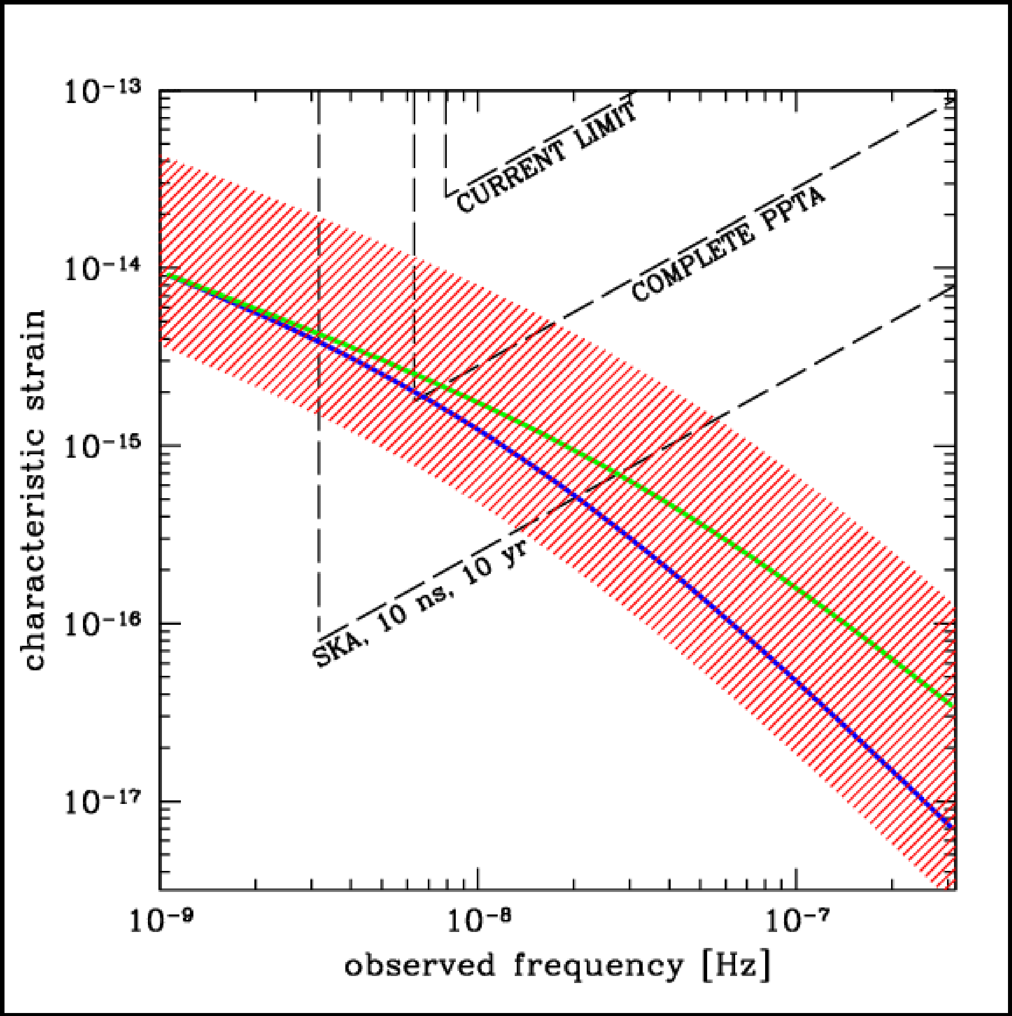
$$h_{\rm eff} = \bar{h}\sqrt{1-Z}\,. \qquad (19)$$

$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M}\, \frac{d^3N}{dz\, d\mathcal{M}\, d\ln f_r} h^2(f_r)$$



Expected background level

(Sesana, Vecchio & Colacino 2008, MNRAS, 390, 192)



Three parameter fit to the background

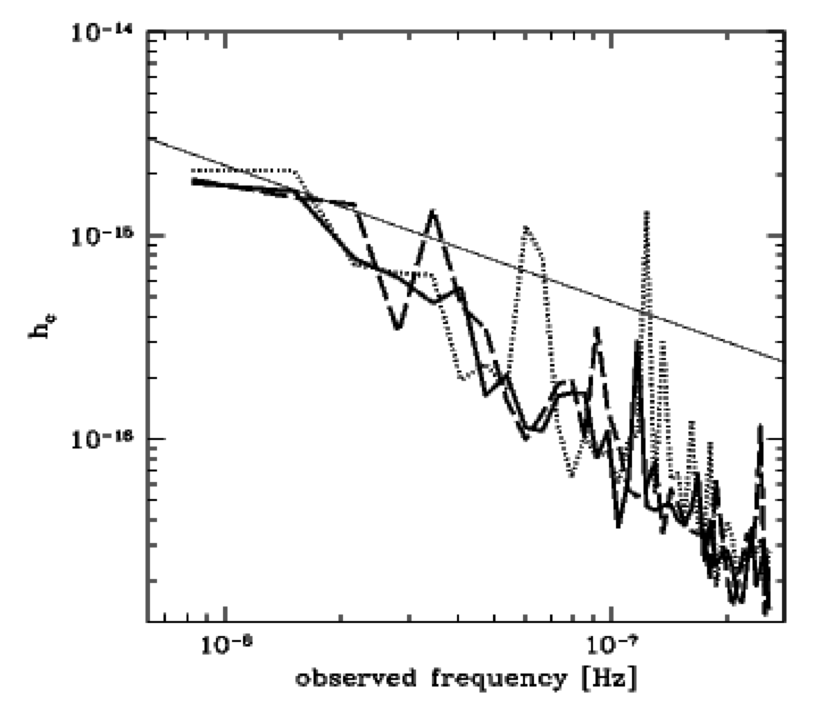
$$h_c(f) = h_0 \left(\frac{f}{f_0}\right)^{-2/3} \left(1 + \frac{f}{f_0}\right)^{\gamma}$$

$$h_0 = (1.93 \pm 1.25) \times 10^{-15}\,,$$

$$f_0 = 3.72^{+1.52}_{-1.30} \times 10^{-8}\,\mathrm{Hz}\,,$$

$$\gamma = -1.08^{+0.03}_{-0.04}\,;$$

Resolvable sources

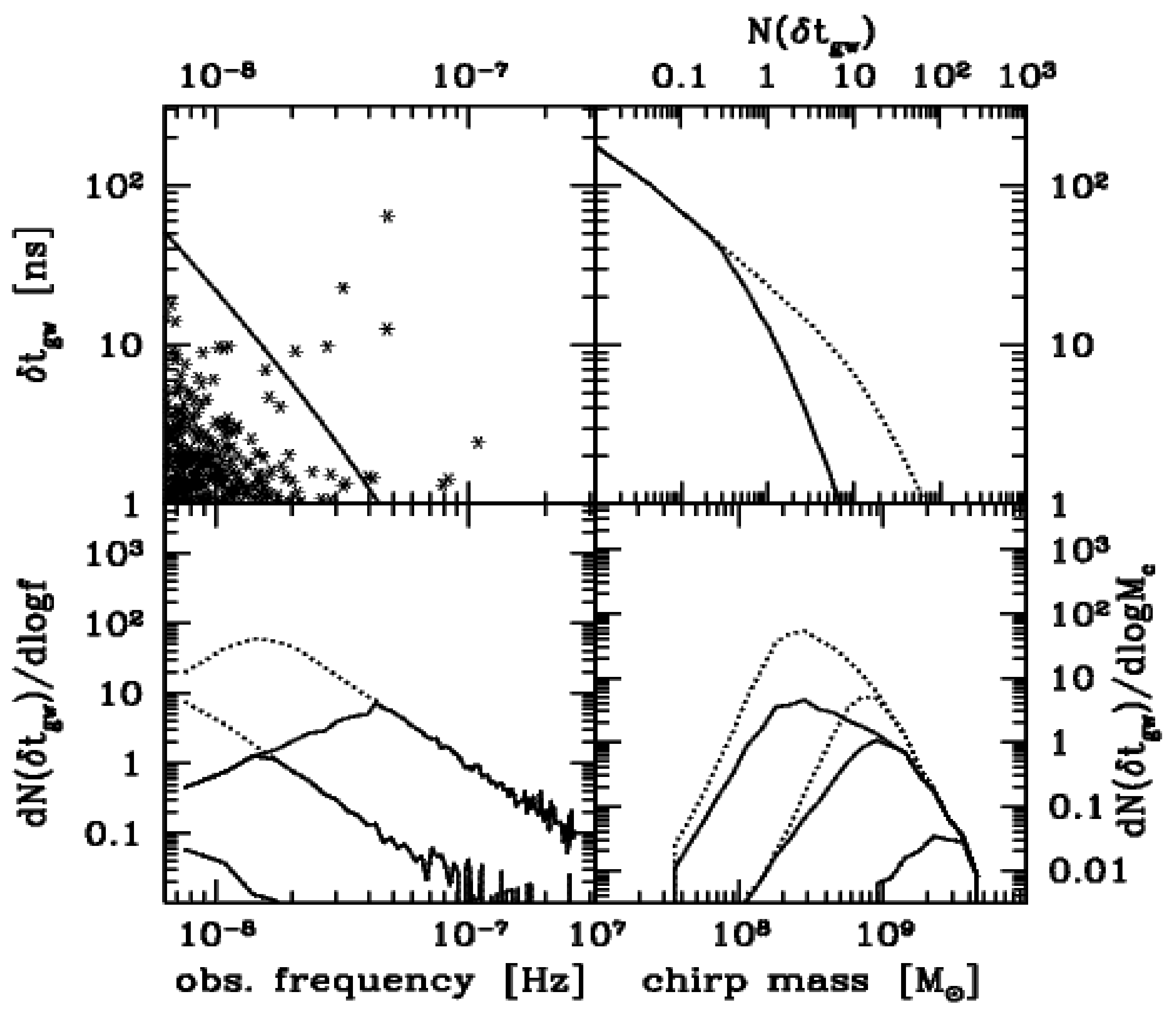


- Nearby and/or massive binaries could rise above the background
- We quantify their statistics and the typical induced residuals to asses required timing precision for detection

$$r(t) = \frac{1}{2}(1+\cos\mu)\left[r_+(t)\cos(2\psi) + r_\times(t)\sin(2\psi)\,,\right]$$

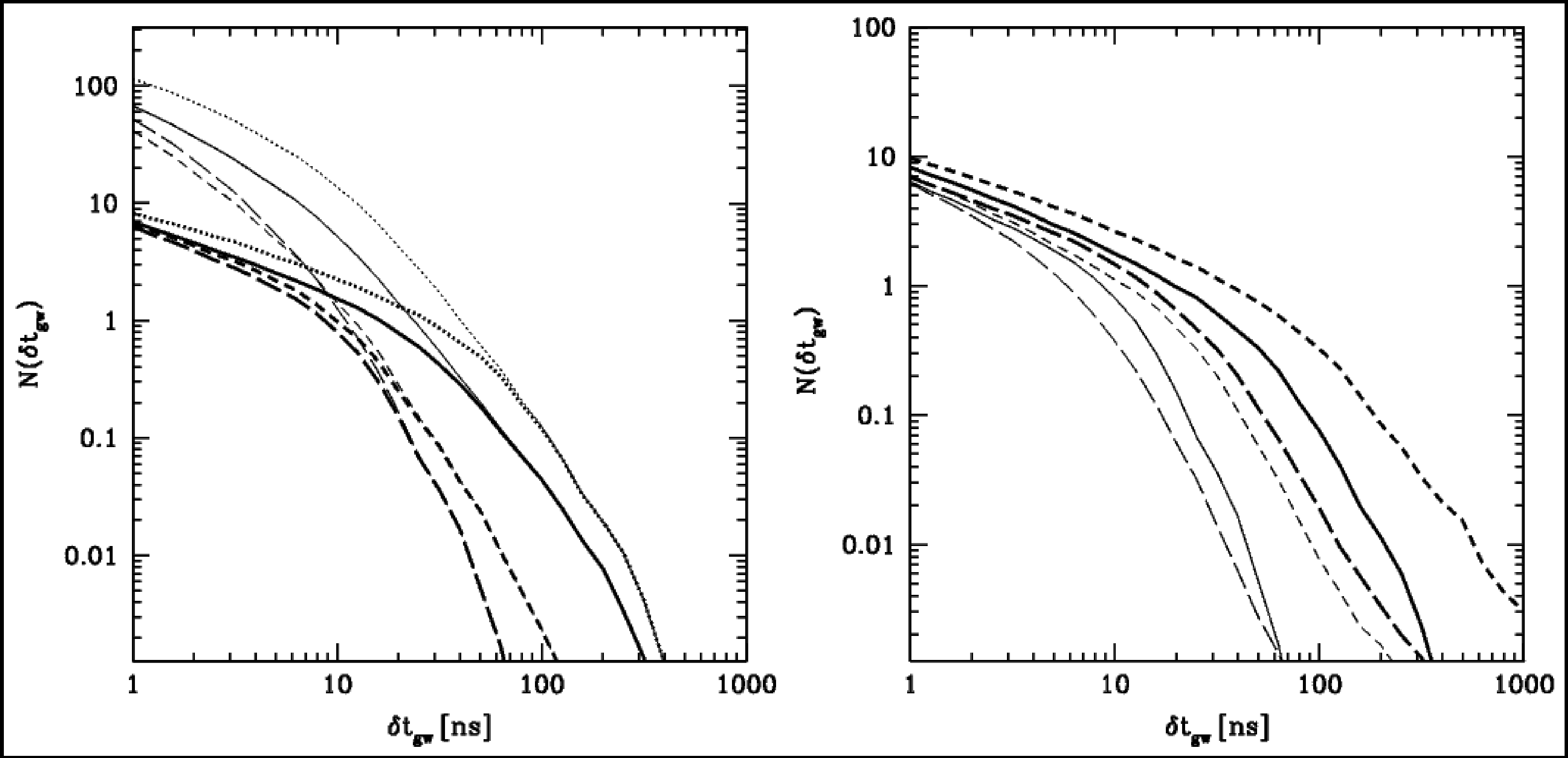
$$\delta t_{\rm gw}(f) = \frac{8}{15}\alpha(f)\sqrt{fT}$$

The typical situation...



Cumulative number of sources...

(Sesana, Vecchio & Volonteri, 2009, MNRAS, in press)

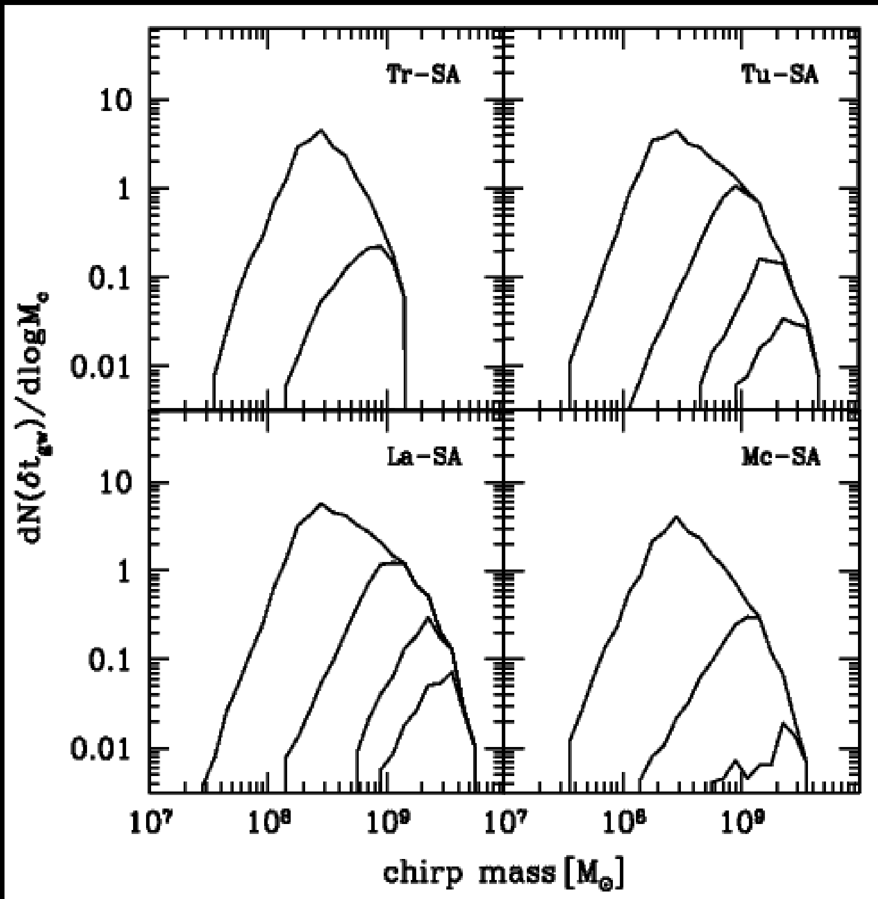


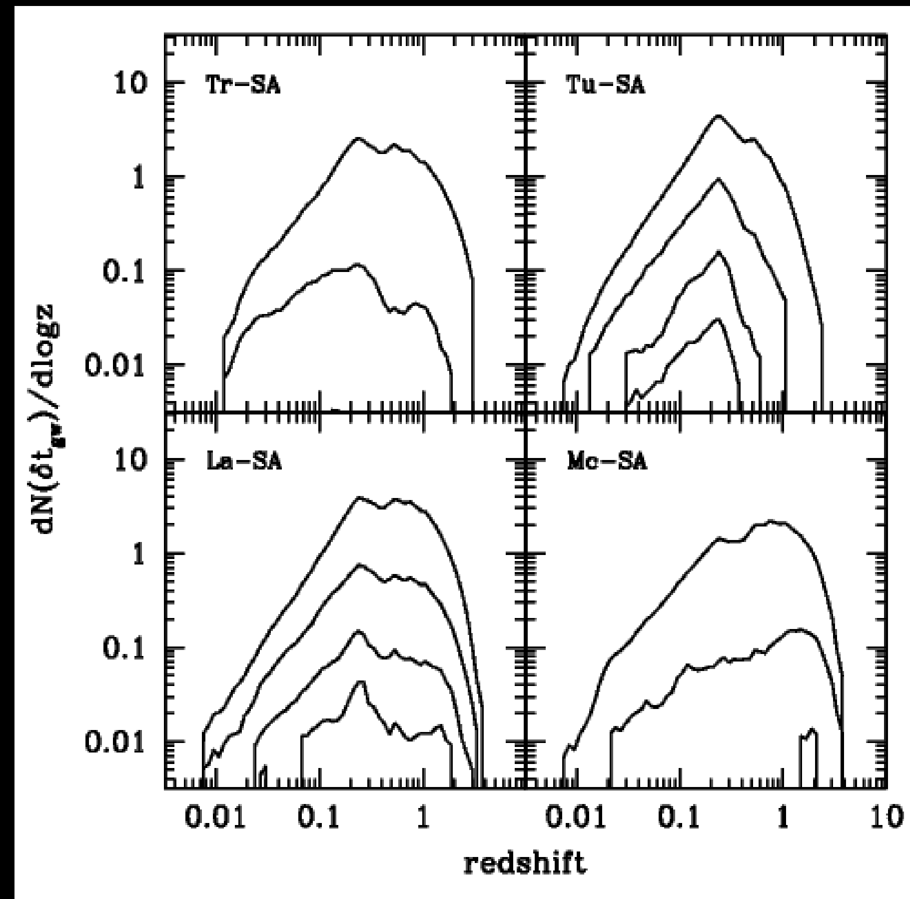
>a total timing precision of ***5-50 ns*** is required to detect an individual resolvable MBHB

>Uncertainties depend on the ***MBH-host*** relation and MBH ***accretion route*** during mergers

Source distributions

Probe MBHs at low-to-medium redshift *0.1<z<1.5*





Probe very massive systems mass > *10^8* solar masses

Parameter estimation

(Sesana & Vecchio, in preparation)

$$\frac{\nu(t)-\nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv \sum_A F^A(\hat{\Omega})\Delta h_A(t;\hat{\Omega})$$

$$R(t) = \int_0^T \frac{\nu(t)-\nu_0}{\nu_0}\,dt$$

The signal depends on the 'antenna beam pattern' which is a function of the relative source-pulsar position in the sky and of the source polarization angle

$$F^+(\hat{\Omega}) = \frac{1}{2}\frac{(\hat{m}\cdot\hat{p})^2-(\hat{n}\cdot\hat{p})^2}{1+\hat{\Omega}\cdot\hat{p}}$$

$$F^\times(\hat{\Omega}) = \frac{(\hat{m}\cdot\hat{p})\,(\hat{n}\cdot\hat{p})}{1+\hat{\Omega}\cdot\hat{p}}$$

We restrict our analysis to *circular non evolving sources*, we consider the *Earth term only* in the residual computation

$$
\begin{aligned}
r_\alpha(t) &= R\left[a\,F_\alpha^{+}\,(\sin\Phi(t) - \sin\Phi_0)\right.\\
&\left. - \; b\,F_\alpha^{\times}(\cos\Phi(t) - \cos\Phi_0)\,\right],
\end{aligned}
$$

$$
\begin{aligned}
R &= \frac{A_{\rm gw}}{2\pi f}\\
\Phi(t) &= 2\pi f t + \Phi_0\\
A_{\rm gw} &= 2\frac{\mathcal{M}^{5/3}}{D}\,(\pi f)^{2/3}
\end{aligned}
$$

The signal depends on *seven unknown parameters*

$$\vec{\lambda} = \{R, \theta, \phi, \psi, \iota, f, \Phi_0\}$$

The sky errorbox is computed as

$$\Delta\Omega = 2\pi\sqrt{(\sin\theta\Delta\theta\Delta\phi)^2 - (\sin\theta c^{\theta\phi})^2}$$

The Fisher information matrix formalism

For a given signal which has any functional dependence on N parameters, the FIM is defined as

$$\Gamma_{jk}^{(\alpha)} = (\partial_j r_\alpha | \partial_k r_\alpha)$$

Where the inner product is

$$(u|v) = \frac{2}{S_0} \int_0^T u(t) v(t) dt$$

Considering detection using M pulsars, the total FIM is the sum of the FIMs of each single pulsar

$$\Gamma_{jk} = \sum_{\alpha=1}^{M} \Gamma_{jk}^{(\alpha)}$$

1sigma errors are given by inverting the diagonal elements of the matrix

$$\sigma_k^2 = (\Gamma^{-1})_{kk}$$

The Monte Carlo simulations

We consider several distribution of pulsars:

1- Isotropic distribution

(varying M=3, 5, 10, 20, 50, 100, 200, 500, 1000)

2-Anisotropic (polar) distribution

(varying the sky coverage=0.21, 0.84, 1.84, 3.14, 6.28, 12.56 srad)

3-The Parkes sample of 20 MSP

For any distribution of pulsar we generate a random sample of GW sorces in the sky:

-amplitude normalized to provide a given SNR in the array

-isotropic sky position

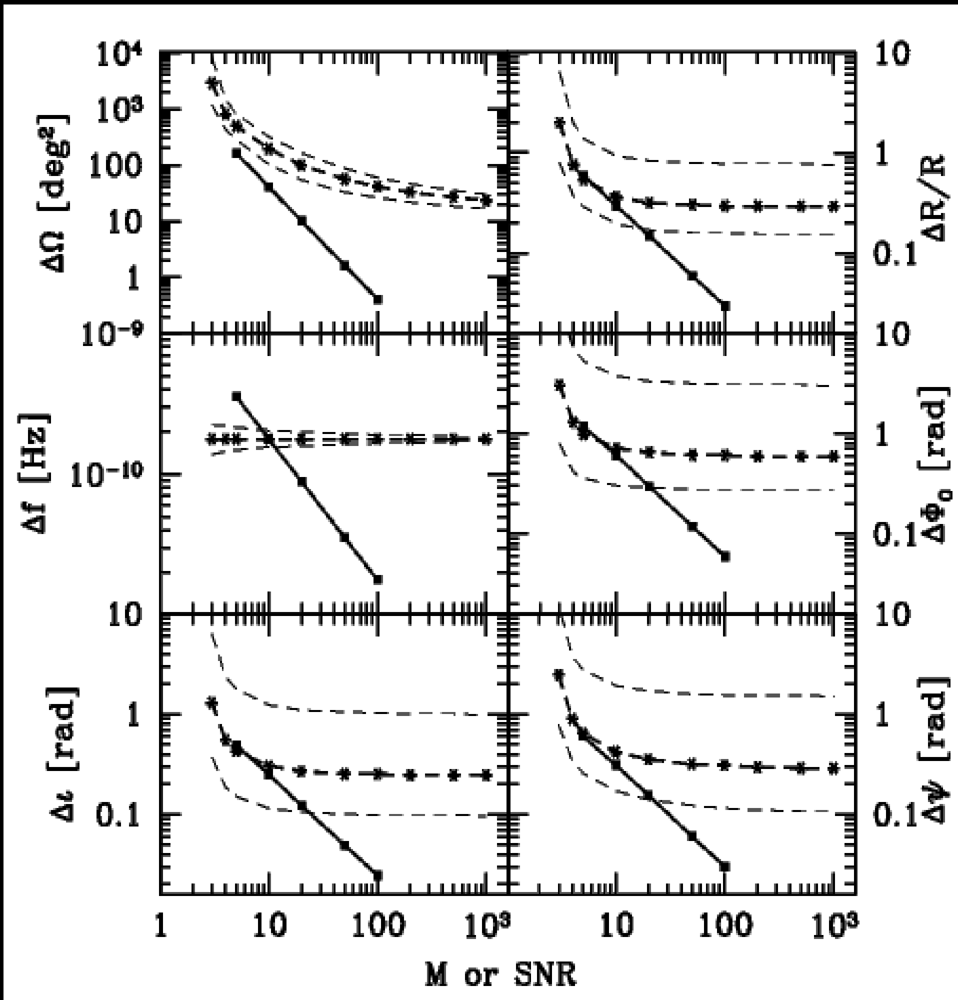
-random phase and polarization

-inclination according to an anisotropic distribution of the orbital L

-random frequency in the range 10^{-8}-10^{-7} Hz

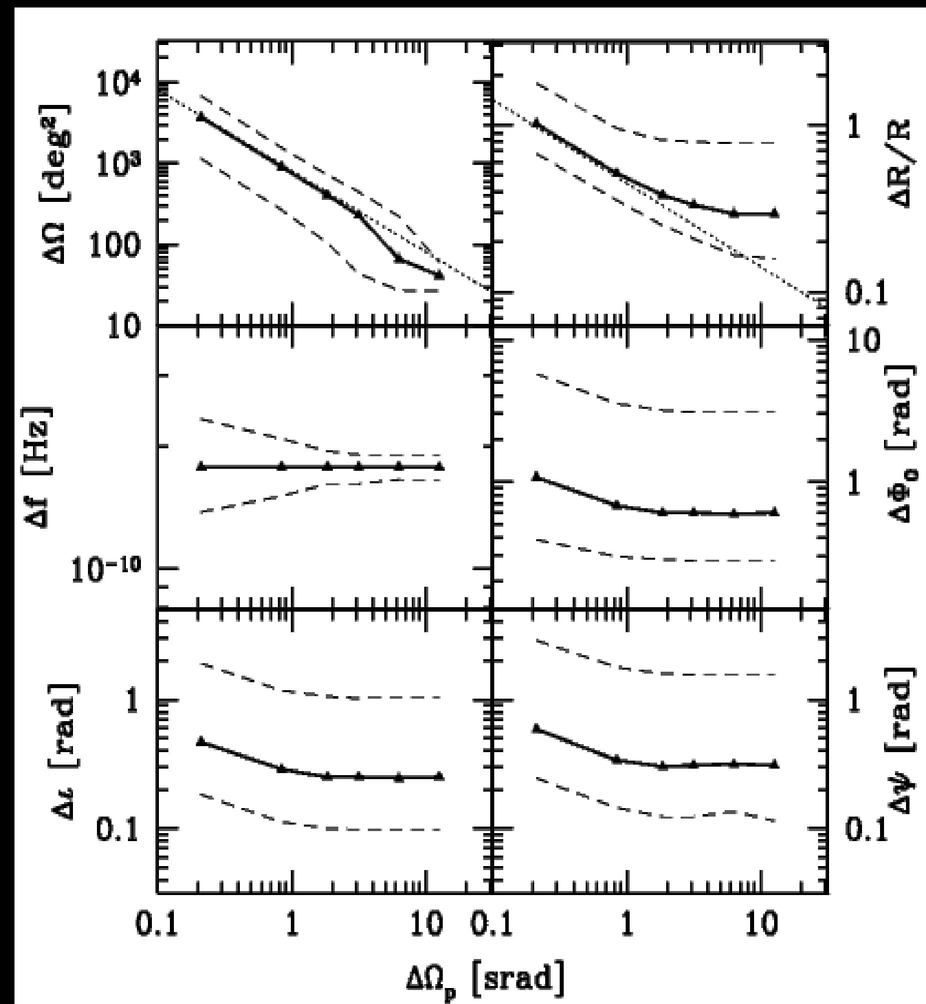
We compute the errors for each source and we evaluate median values, etc etc...

Median 1σ errors

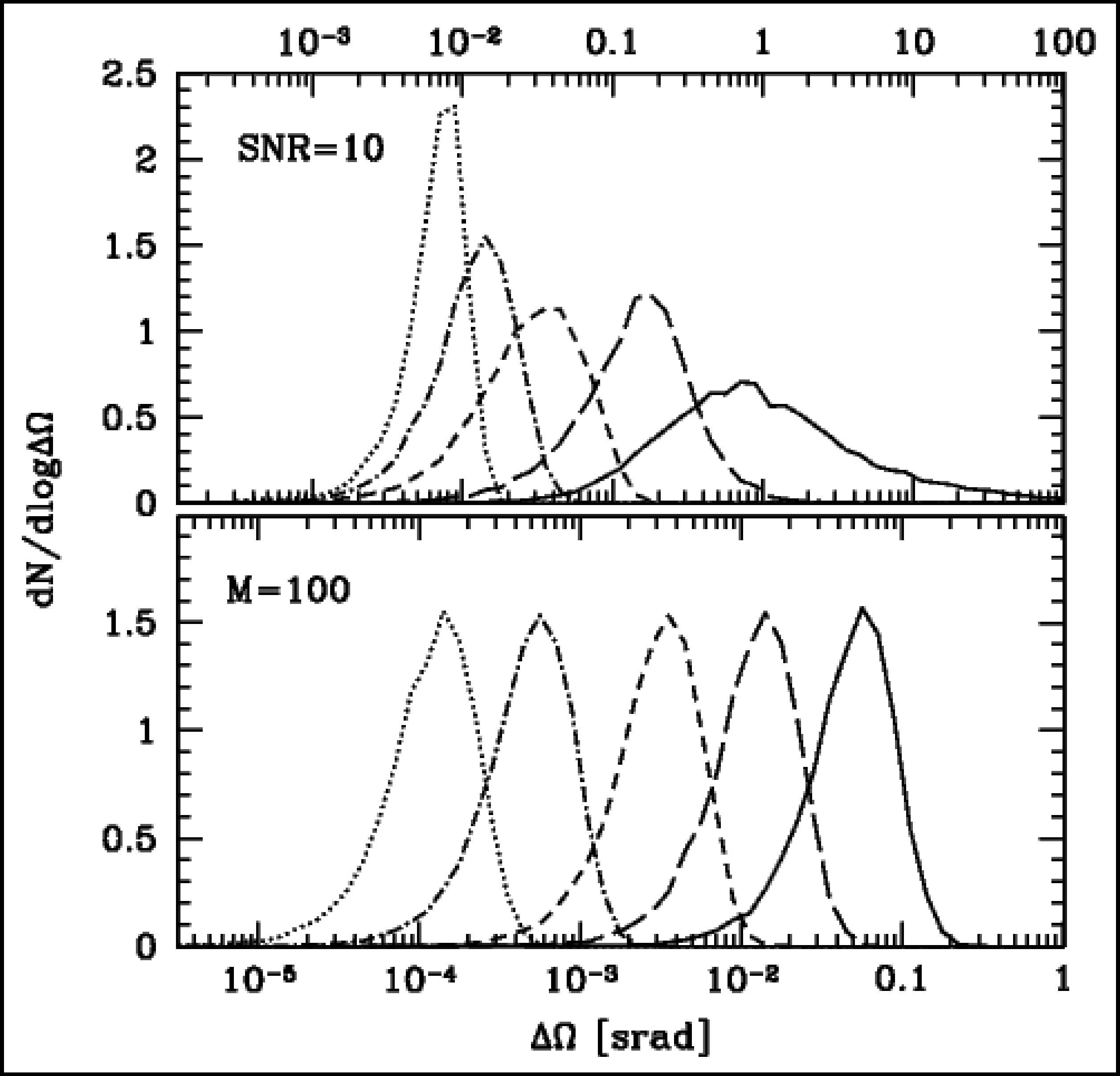


As a function of the number of pulsars and of the SNR

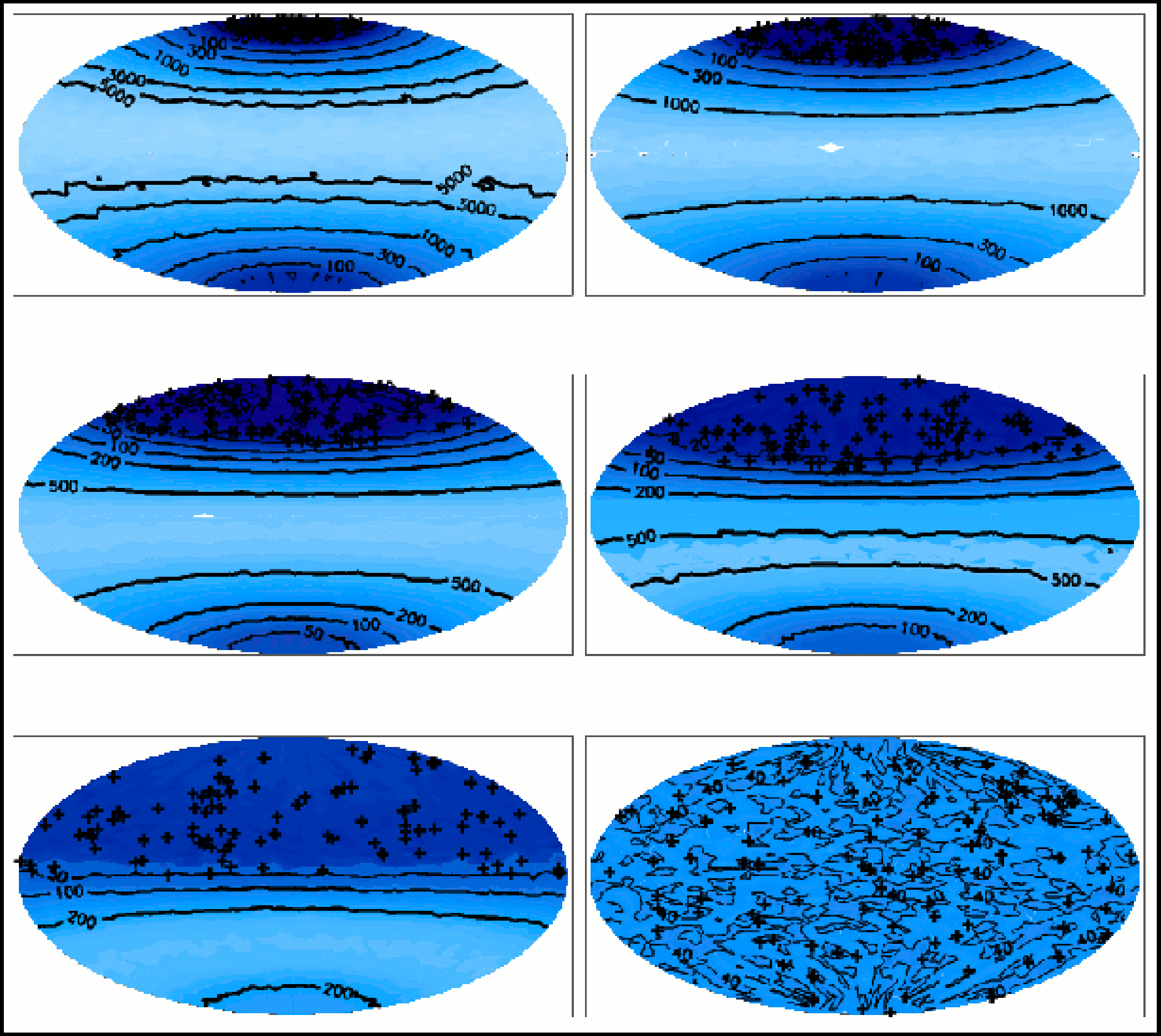
As a function of the sky coverage of the array of pulsars



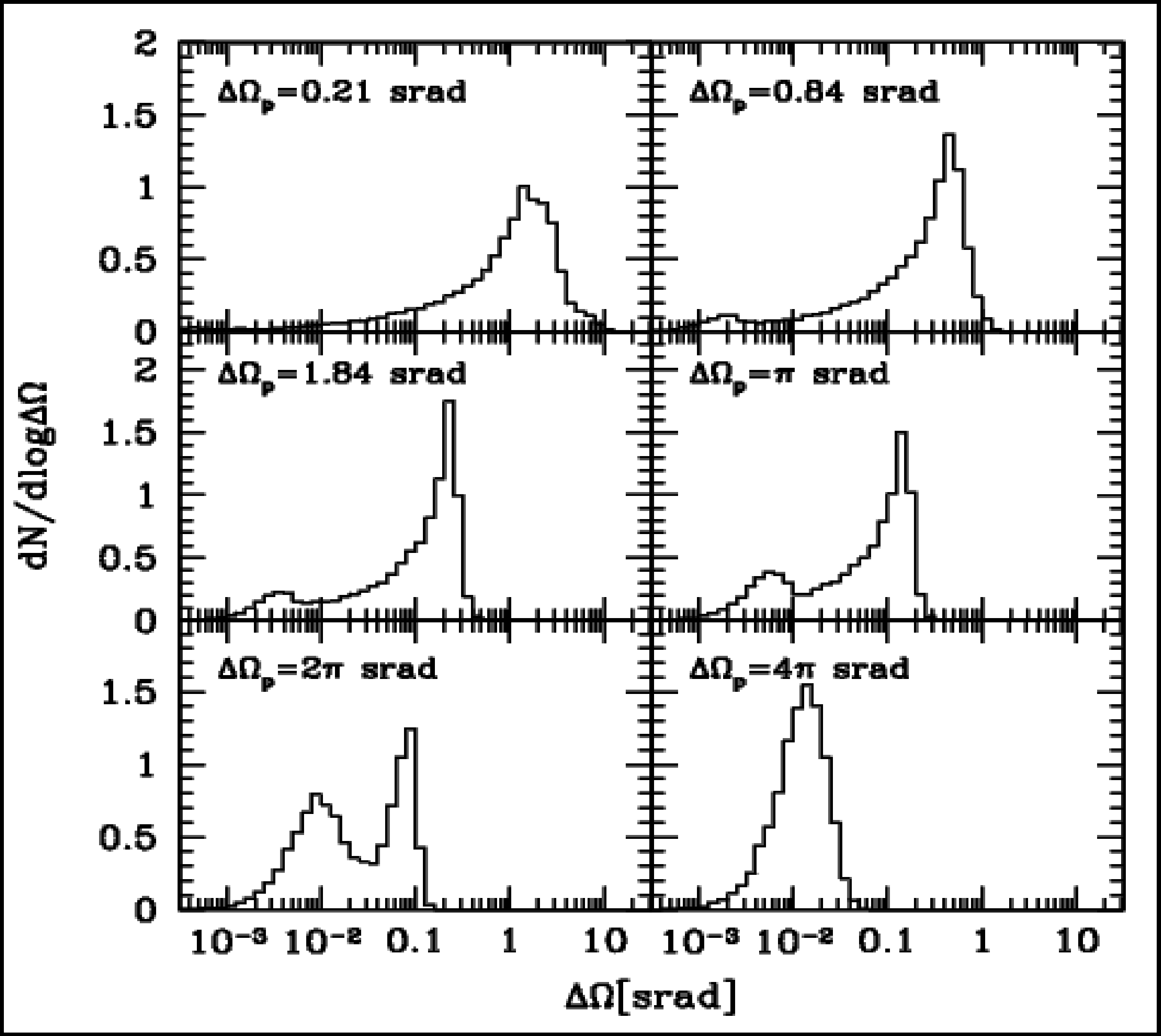
Typical ΔΩ distributions



Anisotropic distribution of pulsars



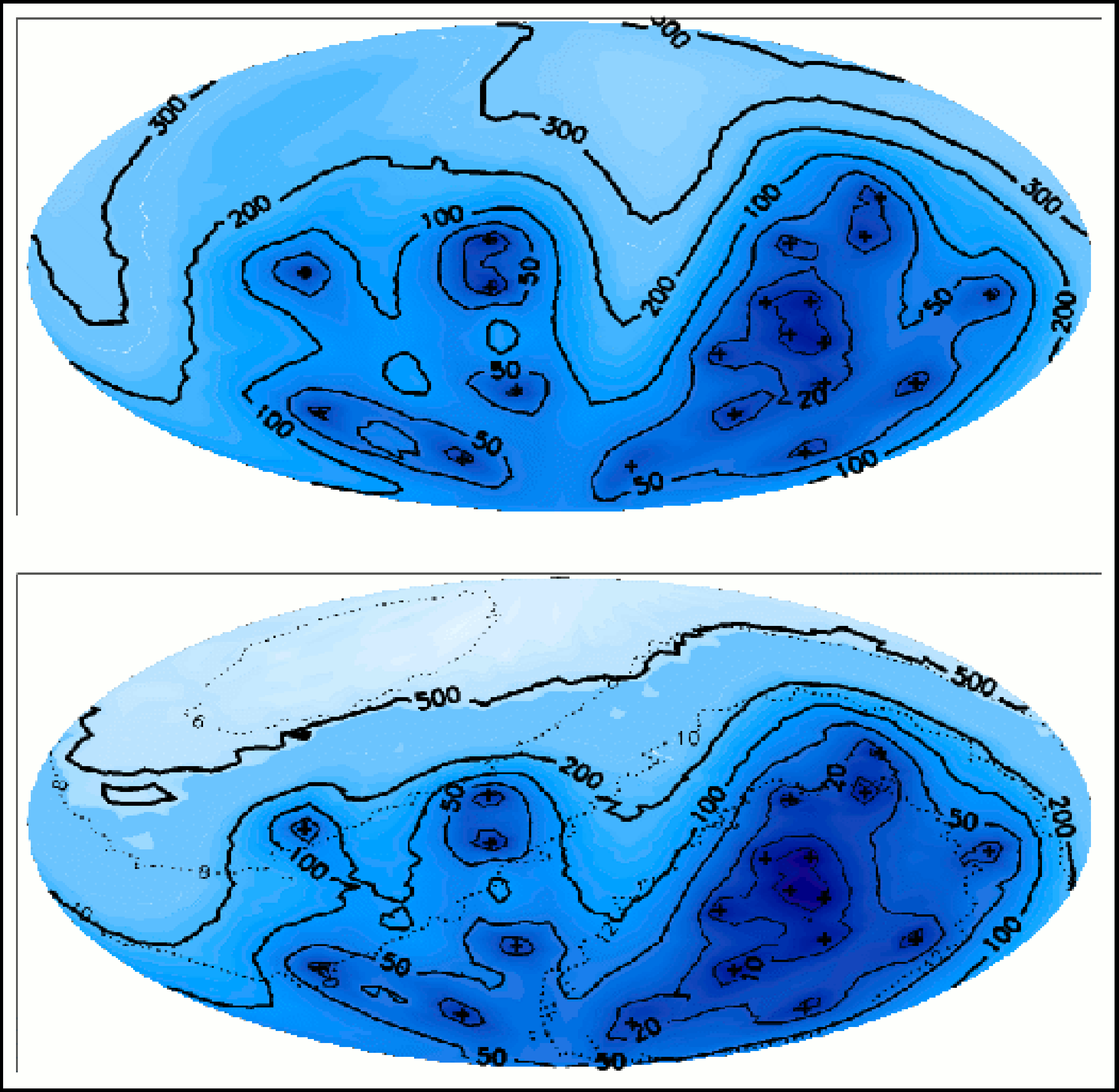
Anisotropic distribution of pulsars



For numbers' lovers

M	$\Delta\Omega_p$[srad]	$\Delta\Omega$ [deg^2]	$\Delta R/R$	$\Delta\iota$ [rad]	$\Delta\psi$ [rad]	$\Delta f/(10^{-10}$Hz)	$\Delta\Phi_0$ [rad]
3	4π	2858^{+5182}_{-1693}	$2.00^{+4.46}_{-1.21}$	$1.29^{+5.02}_{-0.92}$	$2.45^{+9.85}_{-1.67}$	$1.78^{+0.46}_{0.40}$	$3.02^{+16.08}_{-2.23}$
4	4π	804^{+662}_{-370}	$0.76^{+1.19}_{-0.39}$	$0.55^{+1.79}_{-0.36}$	$0.89^{+2.90}_{-0.54}$	$1.78^{+0.41}_{-0.33}$	$1.29^{+5.79}_{-0.88}$
5	4π	495^{+308}_{-216}	$0.54^{+0.84}_{-0.25}$	$0.43^{+1.35}_{-0.28}$	$0.65^{+2.10}_{-0.39}$	$1.78^{+0.36}_{-0.30}$	$0.98^{+4.27}_{-0.62}$
10	4π	193^{+127}_{-92}	$0.36^{+0.57}_{-0.17}$	$0.30^{+0.93}_{-0.19}$	$0.42^{+1.49}_{-0.25}$	$1.78^{+0.26}_{-0.23}$	$0.71^{+3.01}_{-0.41}$
20	4π	$99.1^{+65.3}_{-44.6}$	$0.31^{+0.51}_{-0.15}$	$0.27^{+0.83}_{-0.16}$	$0.35^{+1.34}_{-0.21}$	$1.78^{+0.22}_{-0.20}$	$0.65^{+2.66}_{-0.36}$
50	4π	$55.8^{30.5+}_{-23.0}$	$0.30^{+0.49}_{-0.14}$	$0.25^{+0.80}_{-0.15}$	$0.31^{+1.26}_{-0.19}$	$1.78^{+0.17}_{-0.16}$	$0.60^{+2.56}_{-0.33}$
100	4π	$41.3^{+18.4}_{-15.3}$	$0.29^{+0.48}_{-0.14}$	$0.25^{+0.77}_{-0.15}$	$0.31^{+1.24}_{-0.19}$	$1.78^{+0.13}_{-0.12}$	$0.60^{+2.49}_{-0.33}$
200	4π	$32.8^{+13.5}_{-11.1}$	$0.29^{+0.48}_{-0.14}$	$0.24^{+0.75}_{-0.15}$	$0.29^{+1.21}_{-0.18}$	$1.78^{+0.13}_{-0.12}$	$0.59^{+2.50}_{-0.31}$
500	4π	$26.7^{+8.4}_{-8.2}$	$0.29^{+0.48}_{-0.14}$	$0.24^{+0.75}_{-0.15}$	$0.29^{+1.21}_{-0.18}$	$1.78^{+0.08}_{-0.08}$	$0.59^{+2.50}_{-0.31}$
1000	4π	$23.2^{+6.7}_{-6.8}$	$0.29^{+0.48}_{-0.14}$	$0.24^{+0.73}_{-0.15}$	$0.29^{+1.19}_{-0.18}$	$1.78^{+0.08}_{-0.08}$	$0.59^{+2.36}_{-0.31}$
100	0.21	3675^{+3019}_{-2536}	$1.02^{+0.76}_{-0.34}$	$0.47^{+1.44}_{-0.29}$	$0.59^{+2.29}_{-0.34}$	$1.78^{+0.56}_{-0.40}$	$1.07^{+4.68}_{-0.68}$
100	0.84	902^{+633}_{-635}	$0.51^{+0.44}_{-0.16}$	$0.29^{+0.88}_{-0.18}$	$0.34^{+1.44}_{-0.19}$	$1.78^{+0.31}_{-0.27}$	$0.68^{+2.87}_{-0.38}$
100	1.84	403^{+315}_{-300}	$0.38^{+0.43}_{-0.13}$	$0.25^{+0.80}_{-0.15}$	$0.31^{+1.27}_{-0.18}$	$1.78^{+0.17}_{-0.16}$	$0.60^{+2.56}_{-0.32}$
100	π	227^{+216}_{-184}	$0.33^{+0.46}_{-0.12}$	$0.25^{+0.77}_{-0.15}$	$0.31^{+1.24}_{-0.19}$	$1.78^{+0.13}_{-0.16}$	$0.60^{+2.49}_{-0.33}$
100	2π	$65.6^{+156.2}_{-38.3}$	$0.29^{+0.48}_{-0.13}$	$0.25^{+0.77}_{-0.15}$	$0.31^{+1.24}_{-0.18}$	$1.78^{+0.13}_{-0.12}$	$0.59^{+2.50}_{-0.31}$
100	4π	$41.3^{+18.4}_{-15.3}$	$0.29^{+0.48}_{-0.14}$	$0.25^{+0.77}_{-0.15}$	$0.30^{+1.24}_{-0.19}$	$1.78^{+0.13}_{-0.12}$	$0.60^{+2.49}_{-0.32}$

Sky position accuracy in the Parkes PTA



Summary

> Future PTAs will detect the unresolved MBHB background

> The background spectrum can be fitted by a ***double power law*** with a break around 10^{-8} Hz

> A timing precision between ***5-50 ns*** should guarantee the detection of at least one individual MBHB

> Sources can be locate in the sky within ***40 deg²*** considering an isotropic distribution of ***100 pulsars*** and ***SNR=10***.

Caveats....

We considered here only circular binaries....

-binaries may be highly eccentric
hardening in stellar background --> eccentricity grows
hardening in a gaseous disk--------> who knows....

-each system then emits a whole spectrum of harmonics

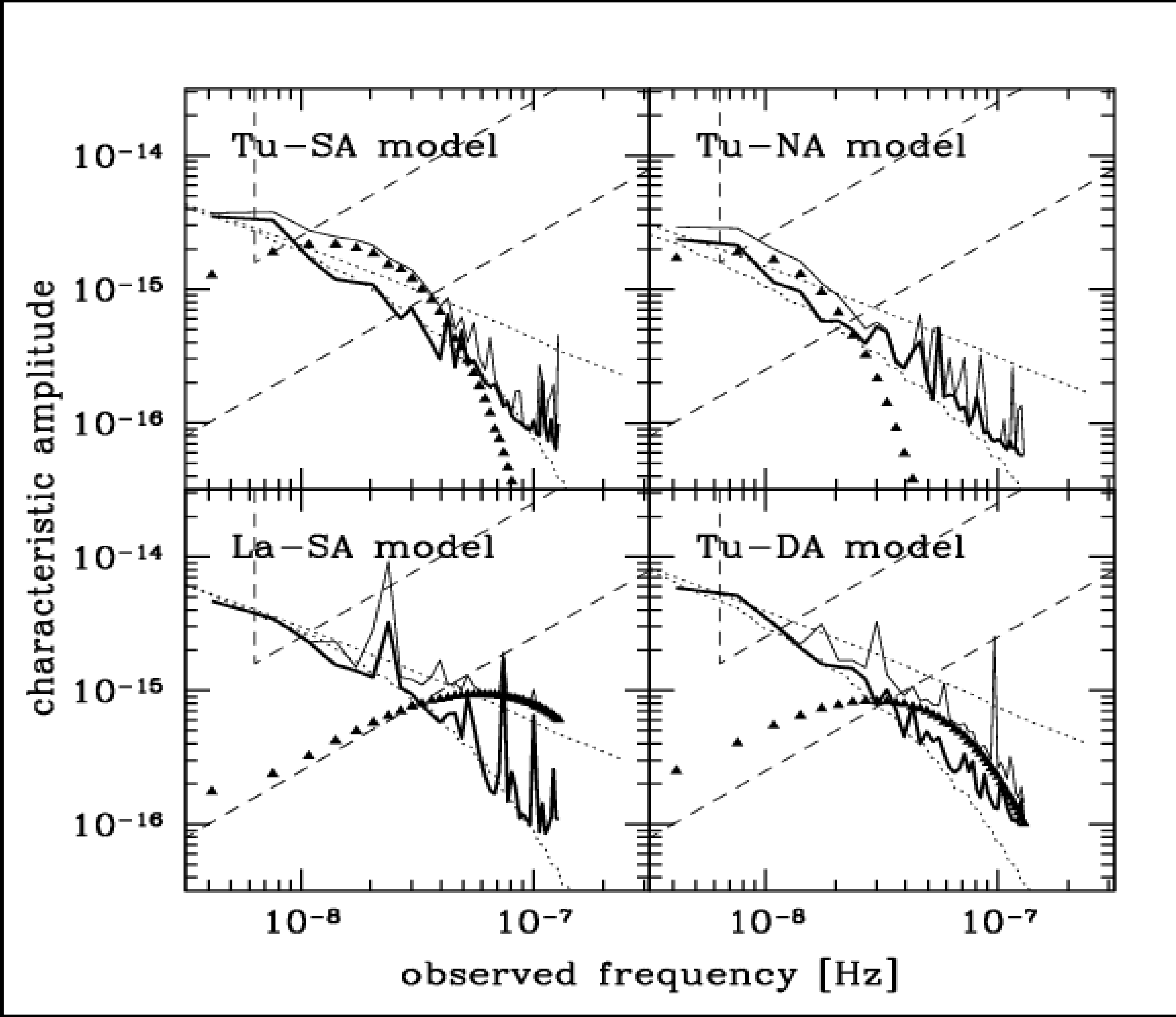
-may have interesting signatures on both the background global shape, and on the characteristics of individually resolvable sources.

We supposed that binaries loose orbital energy by means of gravitational wave emission only....

-extremely inefficient at low frequencies

-an effective shrinking mechanism (timescale ~10^6 yr) may cause the suppression of the low frequency background

Examples, eccentric bursts....



Work in progress

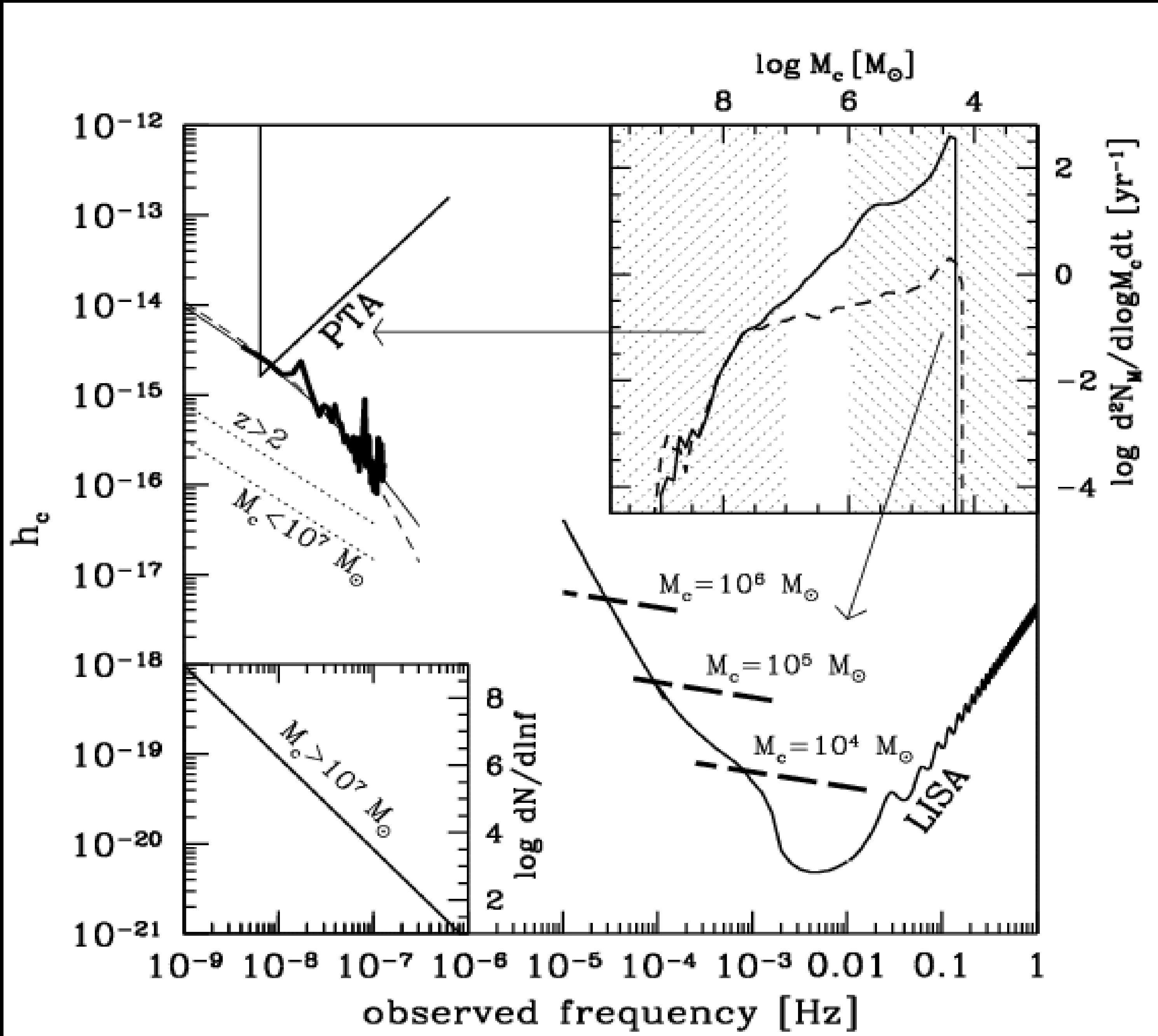
> Self consistent binary evolution including 'large scale' shrinking mechanisms

> Self consistent eccentricity evolution

> Detection and parameter estimation:

-what astrophysics to learn?

-EM counterparts?



log M_c [$M_\odot$]
8
6
4
2
0
−2
−4
log $d^2N_M/d\log M_c dt$ [yr^{-1}]
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
h_c
PTA
z>2
$M_c<10^7\ M_\odot$
$M_c=10^6\ M_\odot$
$M_c=10^5\ M_\odot$
$M_c=10^4\ M_\odot$
LISA
$M_c>10^7\ M_\odot$
8
6
4
2
log dN/dlnf
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 0.01 0.1 1
observed frequency [Hz]

...HIERARCHICAL MBH FORMATION

(Volonteri, Haardt & Madau 2003)

GENERAL FRAMEWORK:

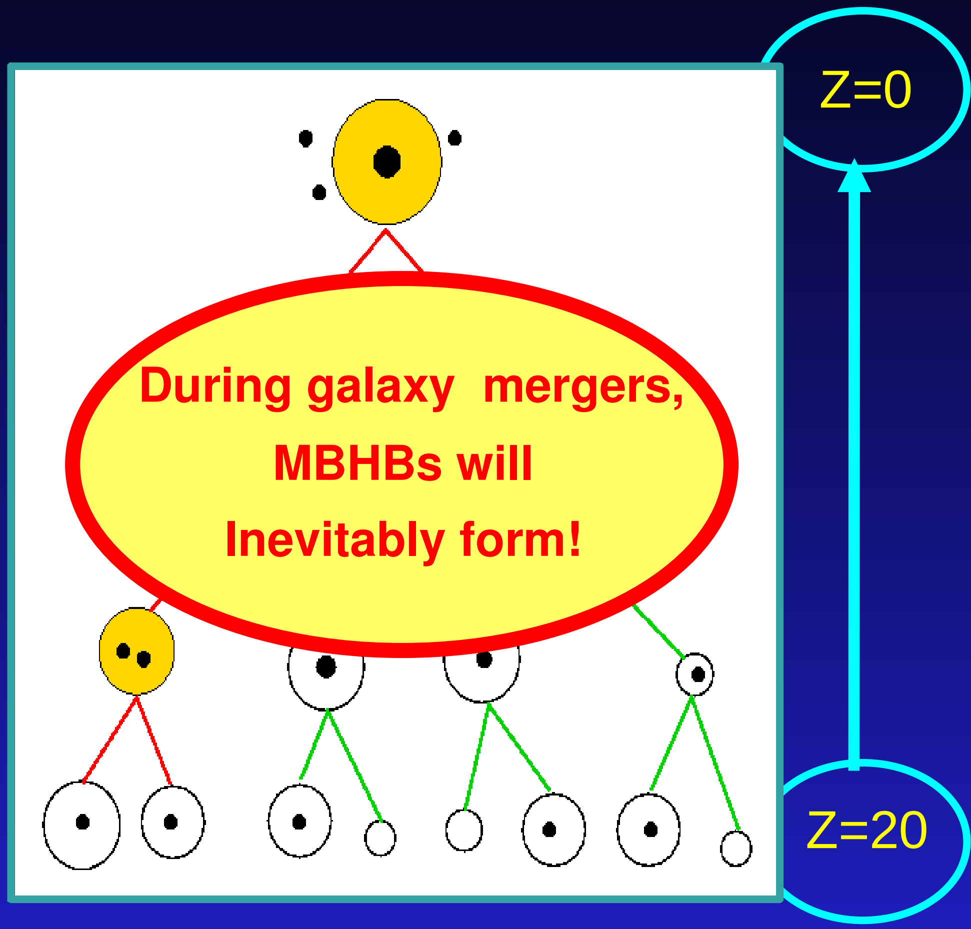
> *We follow backwards the merger hierarchy by means of EPS Monte-Carlo merger tree*

> *Halo density profile:*

- *DM: NFW*
- *Baryons: SIS*

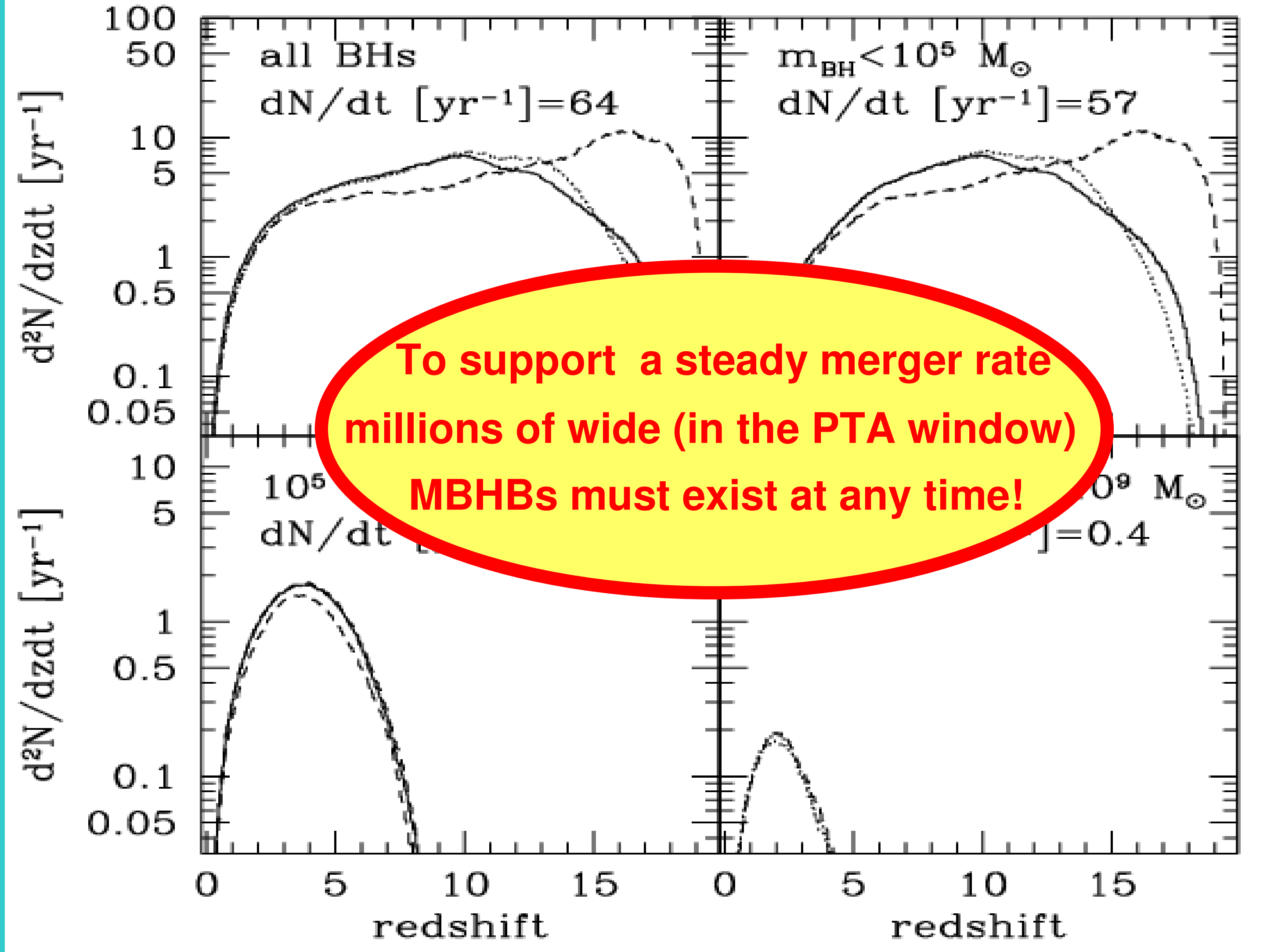
*The semi-analytic code follows the **accretion** and the **dynamical history** of BHs in every single branch of the tree*

*The adopted threshold for density peaks hosting a seed ensures an **occupation fraction of order unity** today for halos more massive than* $10^{11}M_{\odot}$



Binary merger trees starting at z=20
In a ΛCDM cosmology

COALESCENCE RATE



To support a steady merger rate millions of wide (in the PTA window) MBHBs must exist at any time!

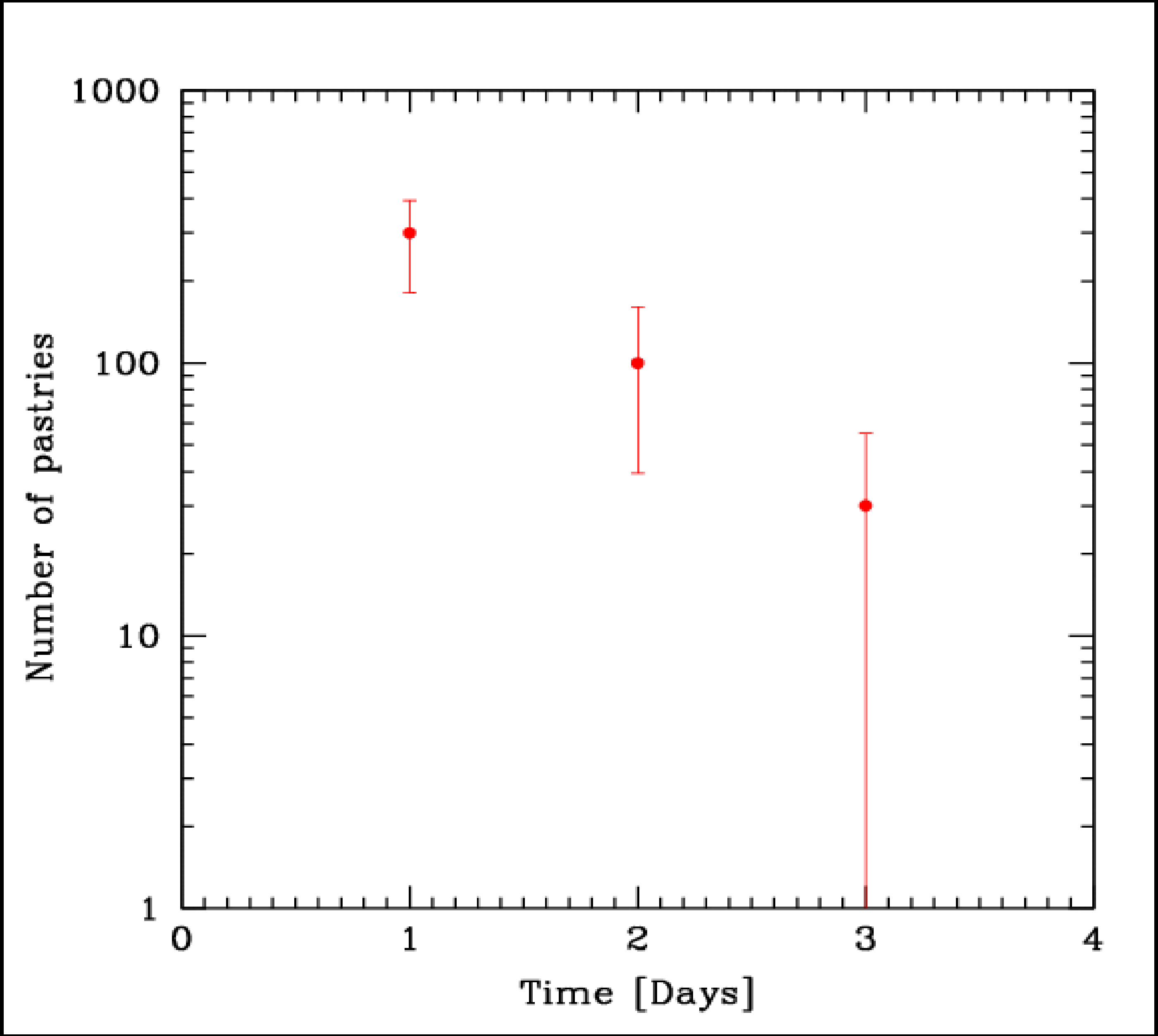
Numbers

Sesana, Vecchio & Volonteri 2008, arXiv:0809.3412

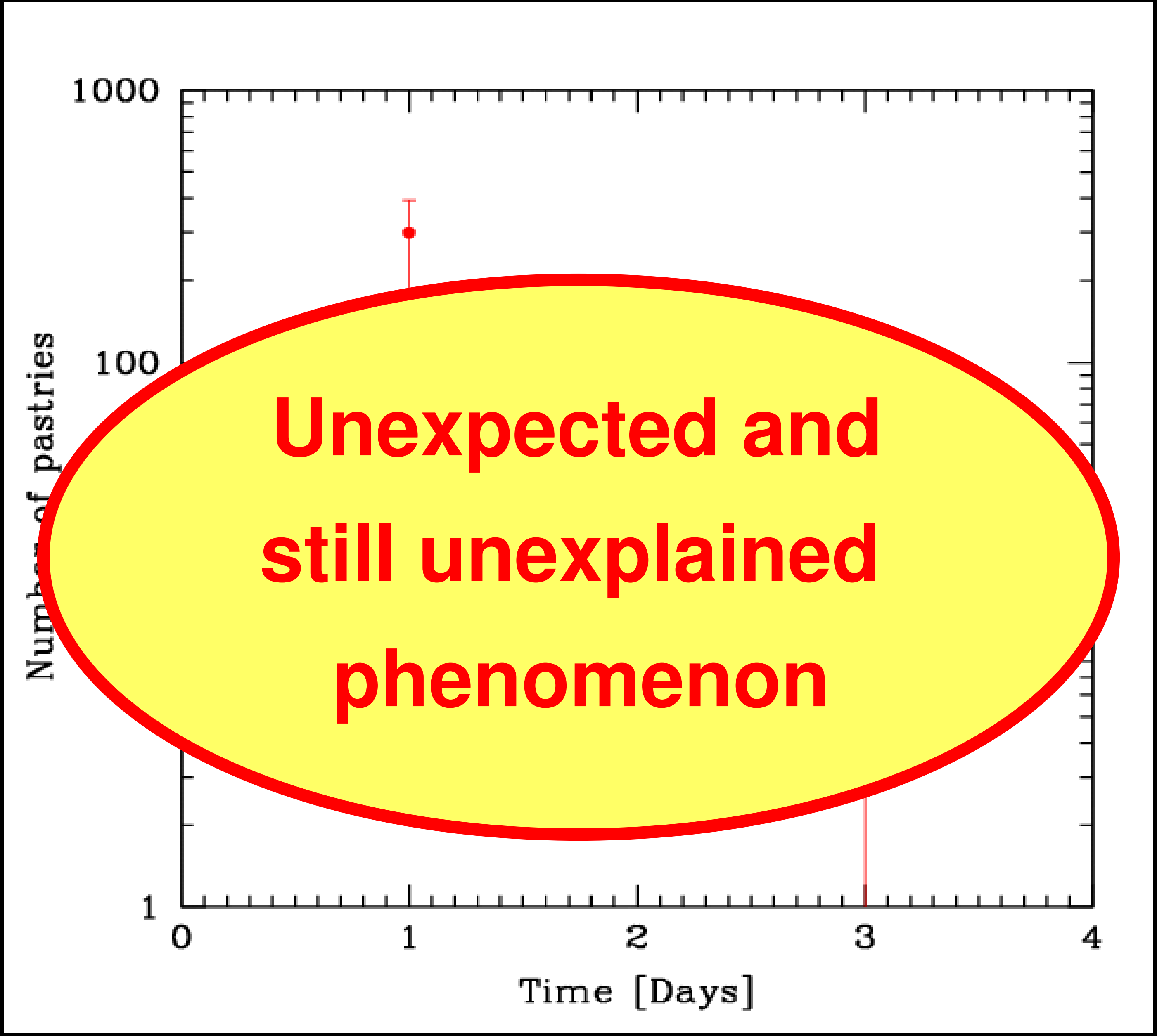
	$M_{\rm BH} - M_{\rm bulge}$ Tundo et (2007)	$M_{\rm BH} - M_{\rm bulge}$ Mclure et al. (2006)	$M_{\rm BH} - M_V$ Lauer et al. (2007)	$M_{\rm BH} - \sigma$ Tremaine et al. (2002)
Single BH accretion	Tu-SA	Mc-SA	La-SA	Tr-SA
	10.9 (3.2)	12.9 (3.4)	15.0 (3.6)	12.6 (3.4)
	1.9 (1.4)	1.9 (1.4)	3.9 (1.9)	1.6 (1.2)
	0.4 (0.6)	0.3 (0.5)	1.0 (1.0)	0.1 (0.3)
	0.2 (0.4)	0.01 (0.3)	0.5 (0.7)	0.02 (0.1)
Double BH accretion	Tu-DA	Mc-DA	La-DA	Tr-DA
	13.8 (3.5)	14.7 (3.5)	17.4 (3.8)	14.8 (3.5)
	3.4 (1.9)	3.2 (1.7)	5.6 (2.3)	2.5 (1.5)
	0.9 (0.9)	0.7 (0.8)	1.9 (1.3)	0.3 (0.5)
	0.5 (0.7)	0.4 (0.6)	1.1 (1.0)	0.07 (0.3)
No accretion (before merger)	Tu-NA	Mc-NA	La-NA	Tr-NA
	8.3 (2.7)	9.8 (3.0)	10.9 (3.2)	8.9 (2.8)
	1.3 (1.1)	1.1 (1.0)	2.2 (1.5)	1.0 (1.0)
	0.3 (0.5)	0.2 (0.4)	0.5 (0.7)	0.05 (0.2)
	0.1 (0.3)	0.005 (0.2)	0.2 (0.4)	0.007 (0.1)

With a 10ns precision *1-to-5 MBHBs* can be individually resolved

SIMPLE EXAMPLE (ongoing experiment)



SIMPLE EXAMPLE (ongoing experiment)



Uncertainties